THE UNIVERSITY OF ALBERTA

AN EXPERIMENTAL COMPARISON OF A TRADITIONAL AND A MODERN COURSE IN ALGEBRA AT THE GRADE TEN LEVEL

by

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The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "An Experimental Comparison of a Traditional and a Modern Course in Algebra at the Grade Ten Level," submitted by Evert Albert Krider in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

This thesis reports the result of an investigation conducted to determine the gains or losses to students which might result from the introduction of a modern mathematics text in the schools of Alberta at the grade ten level.

An experimental group and a control group, each consisting of thirty-five grade ten students were chosen from the Edmonton Public School System. The experimental group was chosen from students enrolled in the Edmonton Public School System's Experimental Course in Grade Ten Algebra. The control group was chosen from regular Algebra Ten classes. The two groups were not significantly different in respect to the means or variances of I.Q. scores, nor the means or variances of the SCAT quantitative scores.

Two teachers, from two different Edmonton Schools, were involved in the experiment. Each teacher instructed one modern and one traditional class.

The control group used a traditional textbook, Mathematics for Canadians by Bowers, Miller and Rourke; the experimental group used a modern text, Algebra One by Hayden and Finan.

The experiment extended over a period of five months. The Cooperative Mathematics Test Algebra I was used as a pre-test and a post-test. The test was divided into two parts--one part to measure understanding and the other part to measure manipulative ability. Thus an understanding, a manipulative and a combined score were obtained.

A t-test was used to test the significance of the difference of

the means at the five per cent level and an F test to test the significance of the difference of variances, also at the five per cent level, in each of the following cases. The means and the variances of the pre-test scores for the two groups were not significantly different. The variances of the post-test scores for the two groups were not significantly different. The difference in the means of the understanding scores and also the difference in the means of the total scores on the post-test were significantly different. The difference favored the experimental group. There was no significant difference in the means of the manipulative scores.

It was concluded that the use of a modern mathematics text at the grade ten level would lead to improved understanding of mathematics and would not result in any losses in manipulative skills.

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CHAPTER I

INTRODUCTION

> A basic step in strengthening the nation's human resources in technology is the improvement of science and mathematics instruction in the elementary and secondary schools.[1]

No one, who is associated with our schools, can be unaware of the recent ferment in mathematics education. There has been an unprecedented surge of effort directed towards the improvement of the mathematics curricula of the schools of the whole western world. Many study groups have been organized to improve the instruction and content of school mathematics. Thousands of schools have been experimenting with new programs in mathematics. Mathematicians, educational leaders, and mathematics teachers are agitating for a 'modern' mathematics course in high schools. They feel certain that an increased understanding of the structure of algebra will allow many facts that would otherwise remain isolated to be learned as elements of a few general principles; algebra would cease to be just the memorizations of algorithms and would become more meaningful to students.

> Mathematics is a lively and exciting subject. New mathematics is created by man with special ability and training. But almost any person--young or old--can understand and enjoy the fascination of mathematical concepts. Unfortunately, school mathematics has not equipped the student to understand the fundamental concepts of mathematics. Mathematics has been taught as a tool subject, and as such, mathematics for its own sake has been neglected.[2]

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[1] Improving Science and Mathematics Education (Washington: The President's Committee on Scientists and Engineers, nd.), p. 1.

[2] Studies in Mathematics Education, A Brief Survey of Improvement Programs for School Mathematics (Chicago: Scott, Foresman and Co., 1959), p. 1.

HISTORICAL SURVEY

Before outlining some of the characteristics of 'modern' mathematics programs and the reasons for the introduction of these programs into our high schools, the writer feels it is appropriate to discuss very briefly the highlights of the events leading up to the recent turning point in the development of high school mathematics curricula in order that the new proposals can be viewed in proper perspective. A brief analysis of the present circumstances to determine the reasons for the sudden change in outlook will better enable the reader to judge the implications of the present 'modern' mathematics movement.

The development of the mathematics curriculum in North America has been closely associated with the changing views of transfer of learning. In the half century before 1900, the theory of mental discipline held sway and it was accepted that transfer took place more or less automatically. Mathematics was of the sequential type, and generally all high school students were required to take it without regard to what practical use it might be put.[3] At this time it was usual for subject matter specialists to determine the content of the mathematics curriculum.[4]

In the first decades of the twentieth century a reaction against the overemphasis on factual knowledge and also the discrediting

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[3]Charles H. Butler and F. Lynwood Wren, The Teaching of Secondary Mathematics (New York: McGraw-Hill Book Company, Inc., 1960), p. 86.

[4]F. G. Robinson, "Change Secondary Mathematics," The B.C. Teacher, XL (February, 1961), p. 219.

of the theory of mental discipline could be noted. The emphasis on specific transfer, the ascendency of pragmatic philosophy, the stimulus-response psychology, and the increased proportion of the population in our secondary schools, all led to more emphasis being put on skills and specific information in mathematics. In the twenties and thirties, the stress was on social adjustment and training for democracy--"preparing the well-informed citizen."[5] As the first half of the century comes to a close the gap between the subject matter specialist and the educationalist was at its widest; and the scholars at the forefront of knowledge started to demand a voice in designing school curricula. Another facet of the development of mathematics that deserves mentioning is the emphasis in the forties on classes for the less gifted and in the fifties on classes for the gifted.[6]

The big turning point in the development of the mathematics curriculum came in the mid-fifties. This resulted from the reaction to the extremes of progressive education, the stimulus-response psychology, the over-emphasis on skills and specific information, the over-emphasis on the social and utilitarian aspect of education, and the extreme negative views on transfer. The following quotation illustrates the changing view on transfer.

> Virtually all the evidence of the last two decades on the nature of learning and transfer has indicated that, while the original theory of formal discipline was poorly stated in terms of the training of faculties, it is a fact that massive general transfer can be

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[5]Chester W. Harris (ed.), "Mathematics," Encyclopedia of Educational Research (Brett-Macmillan, Ltd., 1960), p. 801.

[6]Ibid.

> achieved by appropriate learning.[7]

The changes in ideas of transfer and the Gestalt psychology gave the reformers the psychological grounds for their movement. Bruner describes the movement in these words:

> What may be emerging as a mark of our generation is a widespread renewal of concern for the quality and intellectual aims of education--but without the abandonment of the ideal that education should serve as a means of training well balanced citizens for democracy.[8]

With this movement the subject matter specialist again became influential.

> Curriculum programs such as SMSG and UICSM sprang from the dissatisfaction of the subject specialists with the preparation being given for their discipline in the schools.[9]

Although this turning point seems to have taken place suddenly about 1954, the proponents of the need for radical change in emphasis were actively campaigning long before this. Professor Cecil B. Read, of Wichita University gave a list of quotations, all taken from articles written between 1917 and 1932, registering complaints voicing the same discontent as that voiced by the "revolutionists" of the fifties.[10] Why did these people suddenly become the authorities in the field of curriculum building? First, the gap between what was taught in schools and what was known in the field became acute because of the explosion of

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[7]J. S. Bruner, _The Process of Education_ (Harvard University Press, 1960), p. 6.

[8]_Ibid_., p. 1.

[9]Phillips Hughes, "Notes on Decisions and Curriculum Design," _Educational Theory_, XII (July, 1962), p. 189.

[10]Cecil B. Read, "What's Wrong with Mathematics," _School Science and Mathematics_, LVIII (March, 1958), pp. 181-86.

knowledge. Secondly, the shortage of scientists and mathematicians came to the public's attention with the first sputnik. With the millions of dollars poured into the cause by the American Government the reformers were away. A number of professional groups, attacking the problem of producing a new mathematics curriculum were set up. The three most influential groups are:

> The Commission on Mathematics of the College Entrance Examination Board (usually referred to as the Commission on Mathematics).
>
> The University of Illinois Committee on School Mathematics, headed by Professor Max Beberman. (Abbreviated UICSM.)
>
> The School Mathematics Study Group headed by Professor Edward G. Begle at Stanford. (Abbreviated SMSG.)

These groups are made up of professional mathematicians, professional educators, psychologists, and usually practising teachers. It is hard to over-emphasize the impact that these three groups have made not only on mathematics curriculum but in the whole spectra of the school curriculum building.[11] One cannot discuss recent mathematics curriculum change without referring to these groups.

In conclusion--at the risk of over simplification--it might be inferred from this brief survey that the development of the mathematics curriculum in North America since the turn of the century has been a series of actions and reactions. If one is to extrapolate from this, it would be expected that a reaction to the modern approach to curriculum building as exemplified by Bruner and the workers in the specific subject matter fields to be discernible. The necessity of research in mathematics curriculum is accentuated by the fact that in the case of mathematics

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[11]Bruner, op. cit., p. 70.

this reaction is not only discernible, but is well established with a substantial following. C. Stanley Ogilvy, Hamilton College, Clinton, New York, writes:

> After twenty years of propaganda in favor of the introduction of New Mathematics, we can now discern the beginning of a swing in the other direction. In almost every new issue of the Mathematics Teacher and The American Mathematics Monthly we find one or two articles cautioning us to move ahead slowly, to guard against discarding good and valuable old material merely to make room for something new for the sake of its newness.[12]

And from a statement signed by sixty-four mathematicians in the United States and Canada:

> Mathematicians, reacting to the dominance of education by professional educators who may have stressed pedagogy at the expense of content, may now stress content at the expense of pedagogy and be equally ineffective.
>
> Mathematicians may unconsciously assume that all young people should like what present day mathematicians like or that only students worth cultivating are those who might become professional mathematicians.[13]

It might be observed, therefore, that although both the "revolutionist" and the "reactionaries" are very vocal, neither have much scientific evidence upon which to base their claims.

THE NEW CURRICULUM

The modern or new mathematics programs have been tried out in many schools and results seem to favor the new programs as compared to the traditional. A brief review of the reasons for the changes should

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[12] C. Stanley Ogilvy, "The Transition from School to College Mathematics," The Mathematics Teacher, LIII (November, 1960), p. 514.

[13] Lars V. Ahlfors, et al., "On the Mathematics Curriculum of High School," Ontario Mathematics Gazette, I (October, 1962), p. 5.

make apparent the necessity of the changes.

Desirability

Different writers give various emphasis to the reasons for the demands for an improved program and also the desirability of a new mathematics program in the high school,[14] but the following list includes the reasons usually given:

1. A better appreciation of the need for scientists and mathematicians in our western world by educators and lay groups alike. The superiority of the Russians in the field of rocketry and their challenge to the Western World and technical productivity has made even the man-in-

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[14]The following references will give the reader a sampling of the reasons found in the literature:

Analysis of Research in the Teaching of Mathematics 1957 and 1958 (Washington: U. S. Office of Education, 1959), p. v.

Leonard Doyal Nelson, "Relations of Textbooks Difficulty to Mathematics Achievement in Junior High School" (unpublished Doctoral Dissertation, University of Minnesota, 1962), p. 1.

Max Beberman, "The Old Mathematics in the New Curriculum," Educational Leadership, LXX (March, 1962), pp. 373-75.

G. Baley Price, "Progress in Mathematics and Its Implication for the Schools," The Revolution in School Mathematics (Washington: National Council of Teachers of Mathematics, 1961), pp. 1-11.

Sister Mary Laurence Huber, S.S., "Development in Mathematics Education at the Junior High School Level Since the Turn of the Century" (unpublished Doctoral Dissertation, University of Buffalo, 1962).

Helen Schneider and Burton W. Jones, The New Mathematics Curricula: What and Why (Washington, D.C.: National Council of Teachers of Mathematics, 1962), pp. 1-2.

Bruce E. Meserve, "New Trends in Algebra and Geometry," The Mathematics Teacher, LV (October, 1962), pp. 453-57.

the-street aware that our schools must offer the best possible program in mathematics to our future scientific personnel.

2. A need for more mathematics for students in areas that formerly made little use of mathematics has been created by the explosive growth in technology and attendent changes in sociological conditions. Examples are economics, psychology, medicine, nursing, and philosophy. Also, a certain literacy in mathematics is becoming increasingly necessary to any informed citizen of a democracy if he is to vote intelligently, understand what he reads and take an active part in molding our civilization.

3. More mathematics has been created in the twentieth century than in all previous time. This has led to the demand, especially by mathematicians, for the introduction of some of the newer ideas of mathematics into the high schools to bring it up to date and make it a living subject once more. There is "a great hiatus existing between the content of the mathematics curriculum and the spirit and genius of modern mathematical thought,. . ."[15]

4. The Field or Gestalt theory of psychology has largely displaced the Thorndike Connectionist Theory, leading to an emphasis on the understanding of the general principles of mathematics instead of an emphasis on specific applications. The rapidity of change in our society prevents us from knowing what specific applications and methods will be needed a few years hence and makes necessary the directing of more explicit and careful attention to the basic structure of mathematics.

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[15]Charles H. Butler and F. Lynwood Wren, _The Teaching of Secondary Mathematics_ (New York: McGraw-Hill Book Company, Inc., 1960), p. 86.

<u>Characteristics</u>

Some of the characteristics of the new programs have already been mentioned incidently, but a brief summary will be appropriate here. In the literature the newly developed programs are generally referred to as 'modern' mathematics and less often as the 'new' mathematics. The term 'modern mathematics' is somewhat misleading as it suggests that these programs include mainly mathematics that have been recently developed. Although these courses do contain some modern topics it is more a point of view that distinguishes them from the traditional programs.[16,17] 'Modern mathematics' and 'traditional mathematics' will be used in this thesis to describe programs that differ as follows:

1. The traditional programs contain nothing that was developed less than two hundred years ago. The modern mathematics programs include such new topics as set theory, inequalities, laws of logic, vectors, probability and statistics, matrices, mathematical structures such as fields and groups, and order relations in geometry.

2. The modern mathematics programs stress underlying structure of mathematics; the traditional do not.[18]

3. The modern mathematics programs put less emphasis on manipulation of symbols and computation than the traditional.

4. The modern mathematics programs usually emphasize discovery

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[16] <u>Ibid</u>., p. 55. [17] Beberman, <u>op</u>. <u>cit</u>., p. 373.

[18] Bruner, in <u>The Process of Education</u>, states that "Grasping the structure of a subject is understanding it in a way that permits many other things to be related to it meaningfully. To learn structure, in short, is to learn how things are related."

by the student of principles more than the traditional.

5. The modern mathematics programs direct more explicit and careful attention to the deductive process. The traditional programs, with the exception of geometry, do very little in this regard.

6. The modern programs generally insist on more precision in the use of mathematical language.

7. The modern programs introduce some topics at a lower grade level than was the case with the traditional programs.

8. Some of the traditional topics are given less emphasis; some examples are elaborate cases of factoring, solution of oblique triangles, and computation with logarithms.

PROBLEM

Although many educational leaders are proposing the introduction of modern mathematics programs into our schools, little scientifically controlled research has been conducted to ascertain the gains or losses to educational objectives produced by these new programs. It is the purpose of this thesis to determine what gains or losses would accrue to students in computation and understanding from the introduction of a modern algebra textbook. Since "the importance of the role of the textbook in determining the content, organization, and mode of presentation of school mathematics can hardly be over-estimated,"[19] the problem can be considered as one of determining what effects on the achievement of students are produced by the introduction of a modern mathematics course.

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[19]Nelson, op. cit., p. 7.

Significance of the Problem

Since the Mathematics Subcommittee of the Senior High School Curriculum Committee is considering the introduction of a modern mathematics program in senior high school and since there is little research as to the merits and demerits of such a program the need for the research carried out in relation to this thesis is obvious.

In this respect Kellog and Johnson say:

> During the period 1957-1960, a time of ferment in mathematics education, a variety of experiments with new curriculum were conducted at local, state, and national levels. Evaluation of programs is a project for the immediate future. In designing and implementing new curriculums and new approaches it is important that answers from research be used.[20]

Later in the same article they state:

> A variety of new content is being brought into the mathematics curriculum, and the contribution of new topics need to be evaluated as well as those of traditional topics. We have little evidence with which to defend the old or the new. No experimental project has reported data adequate to permit evaluation of its effectiveness.[21]

And finally the following quotation from the School Mathematics Study Group sums up very well the reasons for doing this type of research:

> Any suggested curriculum revision inevitably and properly raises questions as to what losses or gains may result from change. In the case of mathematics courses prepared by the SMSG, exemplifying greater emphasis on mathematical content and understanding, it was conceivable that reduction of time spent on drill might detract from the acquisition of manipulative skill or result in lower scores on conventional achievement tests. The legitimate concern over this expressed by educators, parents, and students has been shared from the outset by all those involved in the work of the SMSG.[22]

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[20]Theodore Kellog and Donovan Johnson, "Mathematics in Secondary Schools," Review of Educational Research, XXXI (June, 1961), p. 272.

[21]Ibid., p. 273.

[22]School Mathematics Study Group, Newsletter No. 10 (November, 1961), Stanford, p. 3.

OUTLINE OF THE EXPERIMENTAL DESIGN

The experimental design and the statistical analysis will be discussed in Chapter III. A brief outline will be given here in order that the reader may obtain an overall view of the thesis.

An experimental group and a control group, each consisting of thirty-five grade ten students were chosen from the Edmonton Public School System. The experimental group was chosen from students enrolled in the Edmonton Public School System's Experimental Course in Grade Ten Algebra. The control group was chosen from regular Algebra Ten classes which were taught by the same teachers as the experimental group. This helped to control the teacher variable. The groups were practically equal in I.Q., in SCAT quantitative scores, and in pre-test scores. The new Cooperative Mathematics Test Algebra I was used for both the pre-test and the post-test. This test was divided into two parts, one to measure understanding, and one to measure manipulative skills.

The control group used the traditional text Mathematics for Canadians by Bowers, Miller, and Rourke and the experimental group used Algebra One by Hayden and Finan, a modern text.

The project extended over a period of five months from the first week in January to the first week in June. At the end of the experiment the post-test was given and the means of the scores determined for both groups for each part of the test and for the total score. The differences of means between the control group and experimental group were checked for significance at the five per cent level with a t-test. Difference in variances of scores were checked for significance both between groups and between pre-test and post-test for the same group.

LIMITATIONS

The research was limited to comparing the achievement of students using a traditional algebra and those using a modern algebra text as measured by a standardized achievement test.[23] The total score on the test was divided into two parts--one a measure of manipulative skill and the other a measure of the students' ability to apply mathematical concepts and "to reason with insight."[24] No attempt was made to determine what procedures or materials of the two texts led to the differences in achievement. Many beneficial outcomes are claimed for modern mathematics courses such as: a better foundation for college mathematics, greater transfer in solving the unpredictable specific problems of the future; and a greater mathematics literacy to enable future citizens to be better informed in our modern world. Of course, no attempt was made to measure these predicted outcomes. The experiment was limited to four classes taught by two teachers in two Edmonton Public High Schools.

OUTLINE OF THE REPORT

The following is an outline of the report of this study:

The present chapter is an introduction and preview of the report.

Chapter II will review the literature related to the problem and its background and will discuss some of the investigations of this and related problems.

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[23]The two texts are compared in Chapter Three of this study.

[24]*Tests, Materials, and Services* (Berkeley, California: Co-operative Tests Division Educational Testing Service, 1962), pp. 22-23.

Chapter III will describe the experimental design of the problem and the method of analyzing the data.

Chapter IV will report the result of the analysis of the data.

Chapter V will summarize the thesis, state the conclusions drawn, and suggest further research.

CHAPTER II

REVIEW OF RELATED LITERATURE AND RESEARCH

INTRODUCTION

"The twentieth century has been the golden age of mathematics, since more mathematics, and more profound mathematics has been created in this period than during all the rest of history."[1]

Not only has much mathematics been created during this century, it has been applied much more widely in more fields of study. Before the turn of the century physicists had already made abundant use of mathematics. In this century mathematics is being used extensively in chemistry and geology. Biology and psychology are becoming more precise sciences, and biologists and psychologists are making more use of mathematics in solving their problems. Even in the social sciences mathematics is being increasingly applied. But it is not only in the sciences that mathematics is finding an ever-widening application; industry and commerce are making increasing use of mathematical methods in solving their problems. And finally, philosophical problems are coming to have a mathematical character. The importance of mathematics in both vocational and cultural training is becoming more apparent each year.[2]

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[1]G. Baley Price, "Progress in Mathematics and Its Implications for the Schools," The Revolution in School Mathematics (Washington: National Council of Teachers of Mathematics, 1961), p. 1.

[2]Irving Adler, "The Changes Taking Place in Mathematics," The Mathematics Teacher, LV (October, 1962), pp. 450-451.

The growing urgency for improving the mathematics curriculum of the high school was finally noted by educators in the early fifties and the challenge for the improvement was accepted by mathematicians, educationalists, classroom teachers, and even psychologists. Little change in either content or method was made in the mathematics curriculum of high schools anywhere in the Western World during the first half of the twentieth century.[3] Now, however, of twenty-one nations reporting to the International Congress of Mathematics at Stockholm, Sweden, August 15, 1962, the vast majority are experimenting with modernized curricula.[4]

Especially in the United States, there has been an ever increasing amount of activity during the last ten years in the field of mathematics curriculum. There are a variety of interested groups and individuals proposing changes. The Scott, Foresman and Company report on thirty recent or current studies in mathematics education.[5]

> Recent efforts to improve the mathematics curriculum in our schools have led to a flood of new texts and new programs and have, of course, raised the questions of how good these are. Opinions are plentiful and strongly held, but little factual evidence is at hand.[6]

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[3]James Henkelman, "Implementing a New Mathematics Curriculum," The Mathematics Teacher, LVI (April, 1963), p. 211.

[4]John G. Kemeny, "Report to the International Congress of Mathematics," The Mathematics Teacher, LVI (February, 1963), p. 66.

[5]Studies in Mathematics Education, A Brief Survey of Improvement Programs for School Mathematics (Chicago: Scott, Foresman and Company, 1959), pp. 1-57.

[6]E. G. Begle, "A Study in Mathematical Abilities," The Mathematics Teacher, LV (December, 1962), p. 648.

This chapter will illustrate, with a few examples, some of these strongly held opinions of both authorities in the field and classroom teachers and also opinions both in favor and opposed to the modern programs. Lastly, it will present a review of the evidence that is available from research related to modern mathematics programs in high school.

AUTHORITATIVE OPINION

Opinions Favoring the Modern Approach

One of the basic characteristics of the new mathematics programs is their emphasis on structure. Bruner says, "Grasping the structure of a subject is understanding it in a way that permits many other things to be related to it meaningfully."[7] More specifically speaking of mathematics Brown says, "The study of the structure of mathematics is the study of the basic principles or properties common to all systems of mathematics."[8] Understanding the structure of mathematics would involve the understanding of the basic underlying general principles so that specifics could be understood in terms of these general principles. According to Bruner[9] such an organization makes it easy to remember and reconstruct material; it narrows the gap between what is taught in school and the frontier of knowledge. It is claimed that understanding the structure

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[7] Jerome S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1961), p. 7.

[8] Kenneth E. Brown, "The Drive to Improve School Mathematics," in The Revolution in School Mathematics, op. cit., p. 23.

[9] Ibid., pp. 23-26.

of a subject will enable a student to apply his general knowledge in specific cases. This will allow students to apply what they learn to problems of the future--problems that cannot now be predicted. Another advantage, such a presentation of mathematics would provide, is real intrinsic motivation, because young people are interested in what they understand. Bruner's famous statement that ". . .any subject can be taught effectively in some intellectually honest form to any child at any stage of development"[10] provides authoritative sanction for the introduction of materials formerly taught only at the university level into high schools, such as the introduction of set theory in the grade ten program.

Price expresses the opinion that, "In general, the content of any subject matter field must be based on an appeal to authority--the authority of leading scholars in that field."[11] It was for this reason that Robert S. Brown[12] reports a project in which questionnaires were sent to forty-nine heads of mathematics departments in the colleges and universities of Ohio. Thirty-two completed questionnaires were returned. Of those answering the questionnaire, 91 per cent believe that work on inequalities should be included in the programs of college preparatory mathematics. More than 50 per cent would like to see number systems, analytic geometry, sets, elementary functions, and probability

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[10]Frank B. Allen (director), "Classroom Experiences with the New Mathematics Programs," in The Revolution in School Mathematics, op. cit., p. 33.

[11]G. Baley Price, "Questions and Answers," in The Revolution in School Mathematics, op. cit., p. 69.

[12]Robert S. Brown, "Survey of Ohio College Opinions with Reference to High School Mathematics Programs," The Mathematics Teacher, LVI (April, 1963), p. 247.

included in these programs.

The original writing of the SMSG material was written by approximately one hundred high school teachers and one hundred mathematicians. It must, therefore, be concluded that the SMSG programs have the approval of many mathematicians. This applies equally well to the other programs.

Many teachers have taught the new course and have reported success. Marlin, a junior high school teacher, speaks of SMSG materials in these words:

> I feel that the students have a better overview of mathematics; they are aware of the fact that mathematics has a structure wherein all is related; nothing exists capriciously; everything is logically arrived at; one thing leads to another; patterns in mathematics can be discovered; mathematics can be exciting; and thinking mathematically can be fun.[13]

<u>The Revolution in School Mathematics</u>[14] discusses some of the reactions of teachers to the new programs. It is stated that teachers with a strong background of mathematics are enthusiastic about the modern programs and that teachers who have taught the improved programs would not return to the traditional texts. Teachers with weak backgrounds have mixed reactions. However, they feel that as they become more experienced they will be able to teach the new materials more effectively than the old. Mention is also made of the enthusiasm of the students and their parents for the new program. Unsigned students' appraisals are given, such as:

> The main purpose of Ball State geometry is not to have us memorize a volume of facts, but to help us learn the processes of reasoning in order that we may be able to answer problems we have never seen

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[13]Lillian Marlin, "SMSG--One Point of View," <u>The Mathematics Teacher</u>, LV (October, 1962), pp. 477-478.

[14]<u>The Revolution in School Mathematics</u> (Washington: National Council of Teachers of Mathematics, 1961), 90 pp.

> before. . . .Although I have found it difficult, I believe that I'm progressing towards the understanding of plane geometry in particular and mathematics in general.

And a comment from a parent, "So different--so interesting--my child never tires of it."

The preceding statements lend support to the belief held by many educationalists that if mathematics is treated as a rational process and the meanings of its processes emphasized it will be learned more readily by students and also remembered longer.[15]

At least these statements do not supply any evidence to the contrary as stated by Mayor: "There is <u>no</u> reliable evidence that the learning of abstract concepts which are interesting and often exciting will <u>not</u> enable the learner better to understand basic concepts."[16]

<u>Action Research</u>

In the foregoing it has been shown that many mathematicians, educationalists, psychologists, teachers, students, and even parents are convinced that the modern mathematics programs are an improvement over the traditional programs. Most of this opinion is not based on controlled research. In fact, the literature indicates that not all educational leaders are convinced that such research is really necessary, in order to arrive at the conclusion that the new programs are better than the old. For example, Mayor and Waetjin in a review of research say:

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[15] Francis J. Mueller, <u>Arithmetic, Its Structure and Concepts</u> (New York: Prentice-Hall, Inc., 1956), p. xiii.

[16] John R. Mayor, "Efforts to Improve Programs and Teaching of Mathematics," <u>The Bulletin of the National Association of Secondary School Principals</u>, XLIII (May, 1959), pp. 10-11.

> All projects in the development of new course materials for mathematics for K through 14 have provided for classroom try-outs of these materials, for reports from teachers who have used materials, and for modification of the materials based on teacher reports. This represents an important and new kind of research in mathematics education. While in most instances it is not controlled research and may not meet certain standards of educational research, nevertheless the practice of trying out new materials in the classroom, with revisions based on try-out experience, is a method with implications for the future curriculum development.[17]

The Encyclopedia of Educational Research also points out that, "In the area of secondary school and college mathematics, the literature is predominantly concerned with observations, and experimental practices by an individual teacher in his classroom."[18] It also refers to the lack of reports of planned research.

Brownell takes an even more extreme view:

> In American educational research with its emphasis on large populations and its excessive reliance upon the control-group technique, it is frequently forgotten that a single instance may be enough to establish something as a fact. . . .Even if what I saw had happened only once and in a single place, the phenomena would still be valid and could justify the drawing of certain conclusions.[19]

These remarks were made in commenting on some arithmetic classes Brownell had observed in Britain. It can be concluded then, that authoritative opinion at present is based mostly on considered personal judgement and large scale try-outs of new materials. A quotation from John Wagner, Assistant to the Director of SMSG, will conclude this argument:

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[17]John Mayor, "Mathematics Education," Educational Leadership, XIX (March, 1962), p. 395.

[18]Chester W. Harris, ed., "Mathematics," Encyclopedia of Educational Research (New York: The Macmillan Company, 1960), p. 799.

[19]William A. Brownell, "Observations of Instruction in Lower-Grade Arithmetic in English and Scottish Schools," The Arithmetic Teacher, VII (April, 1960), p. 173.

> These texts were revised at a writing session at Stanford University in the summer of 1960. The revisions were based upon classroom evidence from the reports of about 100 teachers and 8,500 students. . . .Can these texts be evaluated with respect to development of mathematical skills and problem solving ability? Some statistical information, and a great quantity of anecdotal material indicates that students using the test do about as well in the development of mathematical skills, but do better in problem solving.[19]

Most would agree, then, with Kemeny when he says, in his summary of the reports of twenty-one nations, that "it seems to be a universal experience that attempts to teach selected topics from modern mathematics well, in reasonable quantities, can be highly successful."[20]

There seems to be little doubt that the modern programs can be taught. The professed convictions of educational leaders, of teachers, and of students would indicate this. It might appear that the question of whether or not to teach the modern courses was already settled. However, the fact that modern courses can be taught does not imply that they should be taught nor does it answer the question of what materials should be included and how it should be presented. There is an urgent need for research to answer these questions. As was stated in the introduction, this chapter presents authoritative opinion both in favor and opposed to modern programs. Up to this point the agreements regarding the new courses have been discussed; now a brief account of some of the disagreements will be presented.

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[19] John Wagner, "The Objectives and Activities of the School Mathematics Study Group," The Mathematics Teacher, LII (October, 1960), p. 457.

[20] John G. Kemeny, "Report to the International Congress of Mathematics," The Mathematics Teacher, LVI (February, 1963), p. 66.

Disagreement Among the Revolutionists and Opposition of the Traditionalists

First, some of the points of disagreement among the proponents of the modern mathematics movement--sometimes referred to as the "Revolutionists"--will be presented and then some of the arguments of the opponents of modern mathematics or the "Reactionist."

It appears from the literature that the modern mathematics movement got its first impetus from those now termed the Formalists. This group consists mainly of pure mathematicians. You can characterize the formalist chiefly by his desire to make school mathematics more deductive--that is, present the ideas of geometry and algebra much more rigorously than they are now presented in high school. They would have all high school mathematics arranged in a formal structure of undefined terms, axioms, defined terms and proofs of theorems. In direct contrast to the formalists are another group of modern mathematics enthusiasts, appropriately called the non-formalists. The non-formalists include Bruner's 'intuitors'. These groups have basically little in common except their dislike of traditional mathematics. It seems that their approach to correcting the faults of the traditional mathematics are diametrically opposed. For example, they both declared that there should be a new emphasis on understanding in mathematics, but the methods they would use in ensuring understanding by school children are the exact opposites; in fact, understanding does not mean the same thing to the two groups. Lambert, representing the formalists, claims that if a student can state, in his solution of the problem, the axioms he is

using to solve a problem he understands it.[21] In other words, in order for a student to understand a solution he must be able to develop a rigorous proof of it.

In opposition to this Max Beberman argues: "There is the terrible hazard of thinking that any approach which emphasizes logical explanation leads to understanding."[22]

Kline takes an even more extreme view: ". . .long and numerous theorems and proofs of relatively simple material only obscure the meaning of what is going on."[23] Contrary to Bruner's stress on intuition, most of the modern programs at the high school level emphasize the deductive process of proof far more than the traditional programs. In this regard, Dr. Kline, Director, Division of Electromagnetic Research, criticizes the reform groups:

> In addition to these changes in subject matter, the reformers require that students learn how to derive in accordance with strict logic and in great detail the properties of the various types of numbers, that is, whole numbers, fractions, negative numbers and so forth. In geometry, students are asked to prove from a very limited set of axioms any number of geometrical facts about points, lines planes, triangles and the like, which intuitively are extremely obvious.
>
> All the new material is arranged in a formal structure of definitions, theorems and proofs. The mathematician would say that the emphasis is now placed almost entirely on the deductive structure of mathematics.[24]

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[21]G. H. Lambert, "The Idea Must Precede the Symbol," The ATA Magazine, XLII (March, 1962), p. 52.

[22]Max Beberman, "The Old Mathematics in the New Curriculum," Educational Leadership, XIX (March, 1962), p. 375.

[23]Morris Kline, "Math Teaching Reforms Assailed as Peril to U.S. Scientific Progress," New York University Alumni News (October, 1961).

[24]Ibid.

This dichotomy goes very deep as illustrated by the following quotation from Rees:

> In discussions of this subject we find a sharp difference in the views of able mathematicians. This reflects the concern of some that the trend toward abstraction has gone too far, and the insistence of others that this trend is the essence of the great vitality of present-day mathematics. On one thing, however, mathematicians would probably agree: that there are and have been, at least since the time of Euclid, two antithetical forces at work in mathematics. These may be viewed in the great periods of mathematical development, one of them moving in the direction of "constructive" invention, of directing and motivating intuition, the other adhering to the ideal of precision and rigorous proof that made its appearance in Greek mathematics and has been extensively developed during the nineteenth and twentieth centuries.[25]

Many other areas in which lack of agreement can be found among advocates of Modern Mathematics are related to this dichotomy. A few will serve to illustrate.

1. The general approach or the specific approach. Some new texts introduce the operations with signed numbers by means of definitions (general rule) and proceed to give them practice in using these. No physical examples are mentioned until a chapter later. On the other hand many so-called modern texts introduce the operation of signed numbers by using specific concrete physical examples. Closely related to this controversy is the next one, which is also illustrated by the same example.

2. More concrete or more abstract. Some authorities argue that it is not mathematics unless it is abstract.

3. More geometry or less geometry. The SMSG program has more

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[25] Mina Rees, "The Nature of Mathematics," The Mathematics Teacher, LV (October, 1962), p. 434.

theorems in geometry whereas the Commission on Mathematics recommends fewer.

4. Pure mathematics or applied mathematics. "It is an urgent problem whether secondary education must restrict itself to pure mathematics."[26] Is it the case that if a student studies pure mathematics with understanding he will be able to apply it in concrete situations or is it true that, "What is equally wrong about today's reforms. . .mathematics is being isolated from the physical and social sciences. As a consequence mathematics becomes more meaningless, more pointless and, so, less attractive."[27]

5. New mathematics or new emphasis on the old. The argument for the introduction of new materials is that it is the only way one can bring about a true understanding of the nature of our number system, since students are too intuitively familiar with the old material to see the necessity of a rigorous development.[28] But Kemeny says, "It is my contention that too much has been made of the allegedly new mathematics in the curriculum."[29]

The preceding shows some of the unsettled issues and areas of disagreement among the proponents of modern mathematics. Are there mathematicians that oppose the modern mathematics trend in general? The answer is yes, there are. Reference to the document signed by sixty-four mathematicians out of seventy-five canvassed will make this clear:

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[26]Kemeny, op. cit., p. 76. [27]Kline, op. cit.

[28]Kemeny, op. cit., p. 71. [29]Beberman, op. cit., p. 373.

> It would, however, be a tragedy if the curriculum reform should be misdirected and the golden opportunity wasted. There are, unfortunately, factors and forces in the current scene which may lead us astray.[30]

One other issue, upon which there is considerable disagreement among recent writers, and which is not related to the underlying dichotomy in mathematics, but to the method of arrangement of instructional materials, is that of logical structuring versus psychological structuring. In discussing the importance of patterns of operations upon numbers in structuring a mathematics program, Meserve points out, "The mathematician sees these patterns as displaying the basic nature of the subject. The psychologist sees these patterns as the most useful medium for understanding and learning to use the subject."[31] Van Engen says:

> It is very difficult to make some mathematicians understand that what is important to a learner as a learner may not be important to a mathematician as a mathematician. Mathematical structures, mathematical language, mathematical idioms, and mathematical thought are not necessarily structured so as to be easily learned by adolescents. They are structured so as to be useful to the mathematician as a mathematician. What is important in learning of mathematics frequently is of little importance to the mathematician.[32]

On the other hand we have Bruner's statement, "The intellectual activity anywhere is the same whether at the frontier of knowledge or in a third grade classroom. . . .The difference is in degree not kind."

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[30]Lars V. Ahlfors, et al., "On the Mathematics Curriculum of High School," Ontario Mathematics Gazette, I:2 (October, 1962), p. 5.

[31]Bruce E. Meserve, "New Trends in Algebra and Geometry," The Mathematics Teacher, LV:6 (October, 1962), p. 453

[32]H. Van Engen, "Some Psychological Principles Underlying Math Instruction," School Science and Mathematics, LXI (April, 1961), p. 246.

And finally Hughes tells us that, "It may be that the distinction between logical and psychological ordering is false."[33]

There is obviously not complete agreement among the leaders of educationa as to what constitutes the best program for high school students. Much research is needed before agreement can be reached. It is probable that there can never be ultimate agreement on some of the issues, but other questions can be answered by research. It is such a question that this thesis aspires to answer--that is, do students following a modern program learn to do problems which (1) require computational skills, and (2) which require understanding, as well as students following a traditional program?

Conclusions

To conclude, it was shown in this section that there is a large group of educators, sometimes termed the 'Revolutionists', who, although they differ among themselves on details, are thoroughly convinced that the traditional programs in high school mathematics need to be replaced by a modern program. Many new programs have been written and tried out in the schools, rewritten and again given a try-out in the schools and this continued until they were satisfied with the results. It was shown that many of the Revolutionists feel that this type of action research[34] is sufficient evidence for recommending the use of these programs in the schools, at least until more careful research can be carried out.

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[33] Phillip Hughes, "Notes on Decisions and Curriculum Design," Educational Theory, XII (July, 1962), p. 192.

[34] Studies in Mathematics Education, op. cit., p. 3.

It was also shown that there is a group of Reactionists who are as willing to condemn the modern programs as the Revolutionists are to commend them and with as little concrete evidence to support their views.

As in politics there are all shades of views between the extremes. A large moderate group are deferring their final conclusions, awaiting more positive evidence. For example, Ogilvy cautions us to move ahead slowly, "To guard against discarding good and valuable old material merely to make room for something new for the sake of newness."[35] Merrill expresses doubt that the "new" will correct all the ills of the present.[36] Wilson believes ". . .much experimentation may be necessary in order to develop a program which will be even partially successful."[37] Here again the necessity of research is emphasized.

RELATED RESEARCH

Introduction

In this section a review of the related research will be presented. Nelson says, "Although many experimental programs in school mathematics have been produced by various groups in the past few years very little definitive research was found which attempted to evaluate the effectiveness of these materials."[38]

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[35] C. Stanley Ogilvy, "The Transition From School to College Mathematics," The Mathematics Teacher, LIII (November, 1960), p. 514.

[36] D. M. Merrill, "Second Thoughts on Modernizing the Curriculum," American Mathematics Monthly, LXVII (January, 1960), p. 76.

[37] Jack D. Wilson, "What Mathematics for the Terminal Student," The Mathematics Teacher, LII (November, 1960), p. 522.

[38] Nelson, op. cit., pp. 21-22.

Many other reviews of research make mention of the lack of adequately controlled research; for example, The Encyclopedia of Education Research makes the observation ". . .not many reports of planned research are available."[39] The lack of statistical evidence of the effectiveness of the programs is mentioned in a review by Estes[40] and in another by Kellogg and Johnson.[41] However, a careful examination of (1) mathematical theses available to the investigator, (2) The Education Index, (3) Dissertation Abstracts, (4) Analysis of Research in the Teaching of Mathematics 1957 and 1958, and the same for 1955 and 1956, prepared by the U.S. Office of Education, and (5) Periodicals such as Journal of Experimental Education, Review of Educational Research, School Science and Mathematics, Educational Leadership, and The Mathematics Teacher, was rewarding in turning up some examples of research related to this investigation. But to date, the amount of research is limited and gives only sketchy and isolated answers to the problem of developing a suitable mathematics curriculum.

Only the more recent reseach is discussed here as much of the research before 1950 is no longer believed to be applicable by many authorities. To illustrate this view, the following examples are submitted:

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[39]Chester W. Harris, op. cit., p. 799.

[40]Ronald V. Estes, "A Review of Research Dealing with Current Issues in Mathematics Education," School Science and Mathematics, LXI (November, 1961), p. 630.

[41]Theodore Kellogg and Donovan Johnson, "Mathematics in Secondary Schools," Review of Educational Research, XVIII (June, 1961), p. 273.

Bruner claims, "The longer and more packed the episode, the greater the pay off must be in terms of increased power and understanding."[42] This is plainly in contradiction to the research on length of class periods that claimed to show that the short, often repeated class periods were superior. The research is not considered applicable any longer because it was based on the drill approach to teaching and apparently does not apply when the basic structure is understood. Bruner believes that the type of curriculum he advocates would "allow longer periods without fatigue."[43] Another area in which previous research is likely to be disregarded because it is no longer pertinent is in the area of research related to extrinsic motivation.

> Experience with any part of school curriculum shows us that children enjoy learning when the work makes sense and they are able to know what they're doing. Rather belatedly we are finding out that arithmetic is no exception--that children can and do like arithmetic if we let the whole class in on the mathematical "secrets" that explain why the arithmetic works.[44]

One other example of the obsolescence of past research is the research on grade placement of subject matter. "Before Sputnik" much research seemed to indicate that it was more efficient to delay instruction in many topics of mathematics.[45] It is not necessary to dwell on how the recent curriculum makers have done the opposite, using the argument that better understanding of material, adapting material to

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[42]Bruner, op. cit., p. 51. [43]Ibid.

[44]When Parents Ask About Arithmetic Today (Scarborough, Ontario: W. J. Gage, Ltd.,), p. 14.

[45]C. W. Washburne, "The Work of the Committee of Seven on Grade-placement in Arithmetic," Thirty-eighth Year Book, N.S.S.E. (University of Chicago Press, 1939), pp. 299-324.

Piaget's[46] natural thought processes, and the accelerated development of children today permits ideas to be introduced much earlier.

In addition, since the modern programs have all been developed since 1950, research related to these programs of necessity must be fairly recent.

In the following outline of research it should be noted that the investigator in most cases did not have access to the original theses and dissertations, and the reference to such unpublished materials is based on the Dissertation Abstracts or The Analysis of Research in the Teaching of Mathematics. Some of the research lends support to the claims of the new curriculum builders.

Research Supporting the Modern Approach

Hammer[47] made recordings as students thought aloud as they worked grade nine and ten problems. He found that students were primarily concerned with getting the correct answer and not as concerned with understanding the problem or its solution. They tended to grasp at superficial clues, and to employ the methods they had learned uncritically. This is probably the result of the over-emphasis on manipulation in the traditional program.

Smith describes an experiment to determine the relative importance of maturity, as compared to experience, in acquiring the concept

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[46]Nathan Isaacs, The Growth of Understanding in the Young Child: A Brief Introduction to Piaget's Work (London: The Educational Supply Association Ltd., 1961), pp. 1-42.

[47]Donald E. Hammer, "Penetration of Mathematical Problems by Secondary School Students" (unpublished Doctoral Dissertation, Teachers' College, Columbia University, New York), 1957.

of a limit in calculus. This experiment was conducted with junior and senior high school students, some of whom had had previous instruction on limits. In each case it was found that past experience did produce a difference whereas an added year of maturity did not. It was inferred that "a matter far more important than consideration of pupils' comparative chronological ages is their comparative backgrounds of experience."[48] This experiment, of course, lends support to the modern practice of introducing materials at a lower grade level.

Kellogg and Johnson report on three experiments that support claims of the Moderns. The first two support the claim that second-track mathematics "should vary only in degree not in kind"[49] from the college preparatory mathematics. The third example supports the claim that there has been too much emphasis on the narrow applications of mathematics in the traditional programs.

The following are the reports of the three experiments:

> Burch (1959) compared two groups of students at one high school after two years of study. The students with two years of formal mathematics were superior to the two-year functional mathematics students in every category tested.
>
> . . . .
>
> Renner (1957) tested functional competence among 237 Iowa high school seniors who had taken one year of algebra or general mathematics. Using covariance to control initial difference on the Iowa Tests of Educational Development, he found a significant difference in favor of the algebra group over the general mathematics group.
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> . . . .
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> Zoll investigated the relative merits of varying amounts of application in plane geometry. No significant difference was found

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[48] Lehi T. Smith, "Could We Teach Limits," The Mathematics Teacher, LIV (May, 1961), pp. 344-345.

[49] Jack P. Wilson, "What Mathematics for the Terminal Student," The Mathematics Teacher, LIII (November, 1960), p. 518.

> between classes with varying amounts of application and control classes nor among experimental classes in regard to ability to solve 'originals', knowledge of facts and principles, or ability to apply facts and principles in practical problems.[50]

A technique often stressed in the new programs is the discovery of principles by the students. Schaaf reported that "the use of student discovery procedures in the experimental course significantly improved the students' ability to generalize in both mathematical and non-mathematical situations and at the same time allowed them to gain reasonable mastery of algebraic principles."[51]

Of course, the main emphasis of the modern courses, which differentiate them from the traditional, is their emphasis on techniques for the development of understanding. Shipp conducted an investigation that illustrates the value of an emphasis on meaning in teaching mathematics. The purpose of the experiment was to determine if variation in the class time devoted to developmental activities and to individual practice work had any effect on arithmetic achievement.

Four classes from each of grades four, five, and six were used in the experiment. The first class from each grade devoted 75 per cent of class time to developmental work (work to develop understanding), the second class devoted 60 per cent of class time to developmental work, and the third and fourth classes of each grade devoted 40 and 25 per cent, respectively, to developmental work. Each of the twelve sections

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[50]Theodore E. Kellogg and Donovan A. Johnson, op. cit., pp. 272-283.

[51]Oscar Frederick Schaaf, "Student Discovery of Algebraic Principles as a Means of Developing Ability to Generalize" (unpublished Doctoral Dissertation, Ohio State University, 1954).

were divided into three levels of mental ability. The experiment was of twelve weeks duration. The pre-test and post-test was a standardized achievement test. Changes in (1) arithmetic understanding, (2) computational skill, (3) problem-solving ability, and (4) total achievement were compared by the analysis of covariance technique.

It was found that groups devoting 75 per cent and 60 per cent of class time to developmental work achieved significantly higher total scores than those devoting 40 and 25 per cent of class time to such work. There was also a significant difference in favor of the 75 per cent and 60 per cent group in understanding arithmetic and in computational skill. There was no significant difference in problem solving ability. The results were the same for all ability levels.[52]

The U. S. Office of Education, in its booklet Analysis of Research in the Teaching of Mathematics 1957 and 1958, report the following research showing that students can learn some of the concepts of modern mathematics.

Byrkit developed units on relations, number theory, sets, transformations and semigroups and taught them to eight senior high school students. The investigation "indicated that the learning of certain ideas in number theory were feasible topics for these students."[53]

Roughhead taught a unit to average grade ten students which

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[52]Donald E. Shipp, Jr., "An Experimental Study of Achievement in Arithmetic and Time Allotted to the Development of Meanings and Individual Practice" (unpublished Doctoral Dissertation, Louisiana State University, Baton Rouge, 1958).

[53]Analysis of Research in the Teaching of Mathematics 1957 and 1958 (Washington: The U. S. Office of Education, 1959), pp. 8 and 25.

included an introduction to set theory, Venn diagrams and graphing of inequalities and equations. The experiment extended over a six week period. Evaluation consisted of observation by the teacher and appraisals of the changes in scores on tests of attitudes and knowledge given before and after the six weeks period. The experiment

> showed that the basic notions of set theory, including its terminology and its visualization by Venn diagrams, and the graphing of inequalities were interesting and comprehensible to these students.
>
> . . . .
>
> Greater understanding of basic mathematical concepts can be achieved through the modern topics than through the traditional topics.[54]

Grubb used materials involving the notions of set variable, relations, and function in a grade ten class. He used methods involving teacher-pupil planning and the discovery of generalizations. "The seventeen students made great gains from September to May in their scores on the Langton First Year Algebra Test."[55]

The preceding experiments seem to provide considerable evidence that students can learn and make use of modern concepts of mathematics. It is to be expected that students learn what they are taught if they learn anything.

"The study conducted by Educational Testing Service produced the important, albeit expected, finding that students in SMSG classes learned substantial amounts of mathematics not included in conventional courses."[56] Another experiment that might indicate that it can be

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[54]Ibid., pp. 8 and 42. [55]Ibid.

[56]Roland F. Payette, "Reports on Student Achievement in SMSG Courses," Newsletter No. 10, November, 1961. (Stanford, California: School Mathematics Study Group), p. 3.

expected that students will achieve a certain goal only if a deliberate effort is made to teach specifically for that goal is the experiment reported in Analysis of Research in the Teaching of Mathematics:

> The general principle that objectives are best achieved when procedures are carefully planned seems to be true in the case of teaching for meaning. When computation was taught with special emphasis on meaning, gains in achievement and meaning resulted. When computation was taught without emphasis on meaning little or no gain in meaning was the outcome. Chaboudy found in teaching a unit on graphs that 'steady gains in meaning of graphs occurred only when the unit specially emphasized meaning. It would seem from these studies that careful planning is required if computation is to be taught with meaning.'[57]

There are a few studies where students using modern materials achieved significantly better than students using traditional materials when the two groups' achievements were compared by means of standardized tests. The SMSG Newsletter No. 10 reports on an experiment carried out by the Minnesota National Laboratory at the grade seven level:

> In 1959-60 we had 13 schools with one experimental and one control class taught by the same teacher. The class assignments were made in the spring of 1959, while the random choice of which of the two sections was to be the experimental was made in September, 1959. In only two schools could we arrange for random assignment of pupils to the two sections. Thus, an analysis on the basis of the class as the experimental unit is completely justified, while an analysis on the basis of the pupil as the experimental unit is probably, but not certainly, justified.
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> . . . .
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> In each school neither or both classes were grouped for ability.
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> . . . .
>
> On the post-tests of May 1960 we found significant differences at the one percent level, in favor of the SMSG both in achievement adjusted for pre-test achievement score and in gains in achievement adjusted for pre-test on aptitude. Thus, there is less than one chance in one hundred that the differences as large as those which

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[57] Analysis of Research in the Teaching of Mathematics 1955 and 1956 (Washington: The U.S. Office of Education, 1959), pp. 17 and 34.

> we found could be attributed to chance.
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> . . . .
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> On the retention tests in September 1960 there were still differences in favor of SMSG, but they were no longer significant.[58]

This last result was a disappointment to the proponents of modern mathematics who believed that the emphasis on understanding of structure in the new courses would enable the students to retain the material longer.

Kushta, whose adviser was Maurice Hartung, compared the effectiveness of teaching algebra in the ninth grade by a method involving organization around certain unifying concepts with traditional methods. There were 131 students in each of the control and experimental groups. Each of five groups of two comparable classes were taught by the same teacher--each teacher working in a different school. The experiment extended over a seventeen week period. At the beginning of the experiment, data were collected on each student from the school records and two tests were administered; one was a specially constructed attitude test and the other a device for measuring interest. The post-test was the Seattle Algebra Test for the End of the First Half, a test to measure the understanding of the nature of mathematics. The interest and attitude instruments were also given at the end of the experiment. The control group used a standard traditional text and the experimental group used the specially prepared 'concept' materials.

The findings were:

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[58] Paul Rosenbloom, "Reports on Student Achievement in SMSG Courses," Newsletter No. 10, November, 1961 (Stanford, California: School Mathematics Study Group).

1. There was no statistically significant difference between the control and experimental groups in manipulative skills.

2. There was a significant difference between the two groups in favor of the experimental group in understanding of the nature of mathematics.

3. There was no significant difference between the two groups on the interest and attitude instruments.[59]

This last investigation would seem to indicate that improvement in understanding can be accomplished when a special effort is made to teach for understanding.

Loman compared achievement of students using UICSM with those using a traditional text. He tested (1) the understanding of basic mathematical concepts; (2) mathematical abilities, and (3) manipulative skills. To test for these, he used UICSM pre-test and post-test, Test of Understanding Basic Mathematical Concepts, the Mathematics Test of STEP, and the Cooperative Elementary Algebra Test. The experiment extended over a one year period. The scores of the tests were compared by means of a t test. The groups were divided into upper, middle, and lower one-thirds according to intelligence and a t test for each of the three questions and each of the three intelligence groups (nine tests in all) were carried out. There was a statistically significant difference in understanding of basic mathematical concepts in favor of the group using the UICSM at the upper one-third intelligence level. The

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[59]Nicholas Paul Kushta, "A Comparison of Two Methods of Teaching Algebra in the Ninth Grade" (unpublished Doctoral Dissertation, The University of Chicago, 1958).

differences in the other cases were not significant.[60]

Whereas the above discussed study indicated that the UICSM had certain advantages for the high ability group, the Minnesota National Laboratory found:

> In grades 6 and 9 the SMSG courses gave especially good results for the students in the bottom quartile. Thus in the 9th grade in 1959 we started with about 16.5% in the bottom quartile according to national norms for 9th-graders. The following September less than 10% were in the bottom quartile for 10th-graders, . . ."[61]

Rosenbloom also reports:

> Generally speaking, students in SMSG classes did at least as well as would be expected in achievement. For example, in 1959-60 in grade 11 on the pre-test about 5% of the students were below the median according to national norms for the 11th-graders, but a year later less than 1% scored below the median for 12th graders. Thus about 80% of the students who were initially below the national median were above the national median the following September.[62]

Inconclusive Research

Up to this point the research reported here has been favorable to modern approaches. However, there are many experiments which showed no significant difference between the two approaches and some that even favored the traditional approaches. The next few pages will be devoted to a description of such research.

Nelson reported on the following two investigations where no difference was observed:

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[60] M. La Verne Loman, "An Experimental Evaluation of Two Curriculum Designs for Teaching First Year Algebra in a Ninth Grade Class" (unpublished Doctoral Dissertation, University of Oklahoma, 1961).

[61] Rosenbloom, op. cit., p. 13. [62] Ibid.

Shuff[63] conducted an experiment in the seventh and eighth grades which was designed 'to overcome the objections to the Laboratory's work through the use of experimental controls.' Analysis of variance was used to compare achievement of students using SMSG materials with that of pupils using the conventional text materials in mathematics. No difference of significance were observed which might favor the use of either SMSG or traditional textbook materials.[64]

In a companion study to that conducted by Shuff, Williams compared the mathematics achievement of grade nine pupils who used traditional textbook materials in that grade with mathematics achievement of grade 9 pupils who had used SMSG textbook materials in that grade. Teachers involved taught both traditional and SMSG classes. On the basis of results from STEP Mathematics Test and Coop Elementary Algebra Test, he concluded that traditional text materials appeared to be as effective as SMSG materials in promoting growth in mathematical achievement of high ability ninth grade pupils. The same conclusion was reached with respect to average to high-average ability students.

He also compared the mathematics achievement of high-ability students who had used SMSG materials in both of these grades. Again achievement difference did not seem to favor the use of either the traditional materials or the SMSG materials.[65]

Rosenbloom, in commenting on the research carried out by the Minnesota National Laboratory in respect to the teaching of SMSG materials at the grade nine level, says:

> The results are inconclusive. In 1959-60 there was a marked improvement for students in the bottom quartile, but students in the top quartile did slightly worse than expected. In 1960-61 the students did worse than would be expected.[66]

In the research carried out for SMSG by the Educational Testing Service, two groups of teachers were selected at random from a group of

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[63]Robert Shuff, "A Comparative Study of Achievement in Mathematics at the Seventh and Eighth Grade Levels Under Two Approaches, School Mathematics Study Group and Traditional" (unpublished Doctoral Dissertation, University of Minnesota, 1962).

[64]Nelson, op. cit., p. 28. [65]Ibid., pp. 28-29.

[66]Rosenbloom, op. cit., p. 20.

teachers willing to teach SMSG materials for the first time, one group taught SMSG materials and the other group taught conventional mathematics. There were approximately thirty teachers in each group for each of six grade levels from grade seven to grade twelve. The students involved in the experiments, both those studying traditional and those studying SMSG materials, were administered tests of scholastic aptitude and tests of knowledge of mathematics in the fall of 1960. These students were given common tests of traditional mathematics and SMSG mathematics in the spring of 1961. On the tests of traditional mathematics skills the students of the traditional group at the grade eleven level had an average achievement score of 21.8; the SMSG group an average score of 21.1. The two scores were adjusted for initial differences in aptitude and achievement. There was not a significant difference in the achievement of the two groups.

At the grade twelve level it was also found that there was no statistically significant difference in achievement of the two groups when the two average achievement scores were adjusted for initial differences in aptitude and achievement.[67]

Kenney and Stockton report on an experiment conducted to determine which of three methods was the best to teach percentage. Three groups of students of 165, 133, and 177 students were chosen. They were equated by using the Horn formula. The first group was taught by a method emphasizing drill procedures with reliance on rules and repetition. The second group was taught by a method emphasizing understanding

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[67]Payette, op. cit., pp. 5-7.

and mathematical reasoning. The third group was taught by a composite of the other two methods. Pre-tests and post-tests were given nineteen days apart. No significant difference between the achievement of the groups as a whole was found.[68]

Zoll found no significant difference in (a) knowledge of geometry facts and principles (b) ability to solve original problems, (c) ability to apply facts and principles in practical problems, among three experimental classes which were given a different number of practical problems for homework. Three teachers each taught one control class and one experimental. The classes were equated for intelligence, geometry aptitude, arithmetic competence, and algebraic competence. It was concluded that assigning verbalized problems of applications for homework was not effective in fostering ability to apply geometric principles.[69]

The argument for using more practical problems in the teaching of mathematics receives no support from another investigation reviewed by Nelson.[70] He reports that Kellogg[71] used three classes of randomly selected tenth grade pupils in an attempt to determine the effects on problem solving of three methods of teaching geometry: (1) an inductive-

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[68]Russell A. Kenney and Jesse D. Stockton, "An Experimental Study in Teaching Percentage," _The Arithmetic Teacher_, V (December, 1958), pp. 294-303.

[69]Edward Joseph Zoll, "The Relative Merits of Teaching Plane Geometry with Varying Amounts of Applications" (unpublished Doctoral dissertation, New York University, 1957).

[70]Nelson, _op. cit._, p. 33.

[71]Theodore E. Kellogg, "The Relative Effects of Variations in Pure and Physical Approaches to the Teaching of Euclidean Geometry on Pupils' Problem Solving Ability" (unpublished Doctoral dissertation, University of Minnesota, 1956).

deductive method with a variety of practical applications; (2) an inductive method with no practical applications; and (3) a deductive method with no applications. All three classes were randomly assigned to treatments and were taught by the same teacher. Analysis of variance was used in testing for significance. On the basis of this result no conclusions could be arrived at concerning the best way of teaching geometry.

Research Supporting a Traditional Approach

The first section of this review of research was given to reporting on experiments whose findings were favorable to the modern mathematics movement. The last section was devoted to research which did not favor either the modern or the traditional view. The final section will be given to reviewing a few studies that seem to favor the traditional approach. In the experiment carried out by the Educational Testing Service, which was previously discussed, the outcome was in favor of the traditional curriculum at the seventh-grade level and at the tenth-grade level. At the seventh-grade level, on tests of traditional mathematical skills the traditional group had an average achievement score 2.6 points higher than the SMSG group. This was a statistically significant difference. At the tenth-grade level the difference was even greater--an average achievement score of 20.27 to 14.02 in favor of the traditional group. However, the two scores were not adjusted for initial differences in scholastic aptitude and mathematics achievement. But, "it is highly likely that the observed difference of 6.25 would

have been only slightly altered if the scores had been adjusted."[72] Therefore, there is little doubt that the advantage of the traditional group over the SMSG group is significant.

Modern mathematics programs tend to be more abstract and more deductive than the traditional programs. Therefore, the results of the investigation conducted by Sobel does not lend support to the modern methods, as will be apparent from the following outline.

Sobel[73] compared the effects of teaching algebra by a method described as "abstract, verbalized, and deductive," and a method described as "concrete, non-verbalized, and inductive." There were seven classes in each group and both groups covered the same content. The post test was given after four weeks--the length of the experiment. The achievement of the concrete, non-verbalized, inductive group was significantly higher for students of above average I.Q. (110-115), although there was no significant difference between the two groups for those of average ability.

SUMMARY

Chapter II, first, illustrated the professed convictions of educational leaders. It was shown that some felt it was the prerogative of leading scholars in mathematics to determine the content of the mathematics curriculum--what should be taught--and it was the role of the educationalist and the practicing teacher to determine

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[72] Payette, _op_. _cit_., p. 7.

[73] Max A. Sobel, "A Comparison of Two Methods of Teaching Certain Topics in Ninth Grade Algebra" (unpublished Doctoral dissertation, Columbia University, 1954).

methodology--how it should be taught. Examples of the use of this premise in producing new programs were given. The chapter presented evidence of the enthusiasm of many mathematicians, classroom teachers, and students for the new programs. The investigator attempted an unbiased approach by also pointing out that there were a few who were opposed to the new programs and, also, some disagreement among the supporters of the new programs. It was related that many of the convictions were strongly held and that much of the opinion was based on massive action research--at least the opinion favorable to the modern programs.

Second, the recent research related to the study was reviewed. Again an unbiased presentation was attempted by discussing the research under three headings: (1) research supporting the modern approach; (2) inconclusive research; and (3) research supporting a traditional approach.

The following comments seem descriptive of the research:

1. The amount of well-controlled research is limited; much more is needed.

2. Research has not provided decisive answers.

3. The conclusions drawn from the research must be tentative.[74]

4. The research usually attempted to compare achievement of students in different programs, and in most cases measuring instruments prepared for traditional programs were used.

In this chapter the need for additional research has been

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[74]Rosenbloom, op. cit., p. 26.

emphasized by showing the lack of agreement among authorities and the indecisiveness of research. However, one generalization might be made--a thread seems to run through much of the research. The generalization drawn is that, if a special effort is made to teach for meaning and understanding, a measurable increase in achievement in the realm of meaning and understanding results, but there is no increase in other achievement such as manipulative ability. That is, if emphasis in teaching is on the understanding of structure in an experimental class as compared with a control class, and a test that measures understanding of structure is used to compare achievement, then there will be a significant difference in the achievement in favor of the experimental class, but if a traditional standardized test is used there will be no difference. An oversimplification would be to say that students learn what they are taught.

To illustrate, it was noted that in research conducted by the Educational Testing Service, students in SMSG classes learned substantial amounts of mathematics not included in conventional courses.[75] Smith[76] showed that background was more important than chronological age. Shipp found that when more time was spent on teaching meaning, the students were better at tests of understanding but not tests of problem solving. Byrkit,[78] and Roughead[79] demonstrated that students

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[75]"Reports on Student Achievement in SMSG Courses," op. cit., p. 3.

[76]Smith, loc. cit.   [77]Shipp, loc. cit.

[78]Byrkit, loc. cit.   [79]Roughead, loc. cit.

could learn the modern concepts of mathematics taught to them. Chaboudy[80] found that meaning of graphs were learned only when meaning was specially emphasized in teaching. Kushta,[81] when comparing modern materials with traditional, found a difference in understanding the nature of mathematics but no difference in manipulative skill. Loman[82] arrived at a similar result except there was only improvement in understanding for the upper one-third of the group. Rosenbloom's[83] report showed variance from grade to grade but generally there was no significant difference in achievement when groups were compared with tests covering traditional topics. Taking a cue from these experiments, the present investigation used a test that is more suitable to measuring expected outcomes for both the traditional and modern courses, as will be shown in the next chapter.

One noticeable exception to this generalization is the work of Burch, Renner and Zoll.[84] It would appear that teaching narrow application and practical problems does not aid pupils in applying mathematics to solving that type of problem.

One final comment on the research reviewed. There are many desired outcomes related to the modern programs that cannot be measured by present day measuring instruments. For example, it is impossible to measure future utility, or intellectual and cultural satisfaction, but decisions must be made now and we need as much evidence as possible upon which to base these decisions.

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[80]Chaboudy, _loc. cit._ [81]Kushta, _loc. cit._ [82]Loman, _loc. cit._

[83]Rosenbloom, _loc. cit._ [84]Kellogg and Johnson, _loc. cit._

CHAPTER III

EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

INTRODUCTION

As discussed in the last chapter, there is not complete agreement among leaders in education relative to the content of mathematics curricula in secondary schools, nor are there consistent or complete answers provided by research to curriculum problems. For this reason the present investigation was planned and carried out. The mean scores on the new Cooperative Algebra I test were compared by means of a t-test for an experimental group using a modern text and a control group using a traditional text. The test was divided into questions requiring only manipulative ability and questions requiring interpretation and application of previous learning--which would require understanding in addition to manipulative skill. In this way a measure of understanding as well as manipulative skill was obtained for the two groups. In this chapter the origin of the Edmonton experimental programs in mathematics will be outlined, a comparison of the two texts used in the investigation will be made, information regarding the tests used will be given, the conduct of the experiment will be described, and the statistical procedure discussed.

EDMONTON PUBLIC SCHOOL SYSTEM'S EXPERIMENTAL COURSE IN GRADE X ALGEBRA

Six teachers in four different schools in the Edmonton Public School System were chosen to conduct an experimental course in modern

algebra commencing the first week in January and terminating the middle of June. The teachers were chosen on a basis of expressed interest in the program and qualification to conduct such a program. The teachers chose the classes they would use in the experiment. Each teacher had one experimental class. Two of these six classes along with two traditional classes were used in the present investigation. An outline of the planned experimental course is given in the appendices. However, as it developed, the course followed the first seven chapters of the text, Algebra One by Hayden and Finan,[1] almost exactly in the two classes involved in this investigation. In these two classes at least, the text determined the course. The material of this text, along with that of the traditional text will be outlined in a following section.

THE EXPERIMENT

The investigator received permission from the Edmonton Public School Board to carry out a controlled experimental study to evaluate some of the outcomes of their experimental program in grade ten algebra, which has been outlined above. Of the six classes involved in the Edmonton experimental program, in only two cases did the teacher of the experimental grade ten classes also teach traditional classes in grade ten. It was decided to use only these two classes in order to control the teacher variable in the investigation. The classes were taught by two teachers in two different schools.

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[1]Dunstan Hayden and E. J. Finan, Algebra One (Boston: Allyn and Bacon, Inc., 1961), pp. 1-241.

From these two traditional classes and two modern classes thirty-eight students in the traditional classes and thirty-eight students in the modern classes were found to have the same mean I.Q. to three significant digits.

Because one student discontinued attending school and because all data were not available for some students, the final number had to be reduced to thirty-five in each group. The thirty-five students using the traditional text, of course, was the control group; the thirty-five students using the modern text was the experimental group. These will be so designated in the remainder of the thesis.

It will be noted that the selection of teachers, classes, and students was not random. Randomness of selection had to be sacrificed in order that the experiment could be conducted in a practical classroom situation. Both the control group and the experimental group were somewhat above-average groups (mean I.Q. 117 as compared to 113 for the two schools). The teachers involved probably could not be considered typical high school teachers as they were chosen by the administration of the Edmonton School System to participate in the modern mathematics project partly because of qualifications to conduct the experimental class.

The teachers were asked by the investigator not to use any of the modern material in the traditional class. At the end of the experiment both teachers assured the investigator that there was no carry-over of ideas or materials from one class to the other.

It is assumed that the ability as measured by the Cooperative

Algebra I Test,[2] of the two populations represented by these groups are normally distributed. As will be shown in the next chapter, there was no significant difference between variances of the scores of the two groups on the I.Q., the SCAT, or the three parts of the Cooperative Algebra I Test.

It was assumed that the Hawthorne effect would be insignificant in this experiment. The Educational Testing Service did some research in this respect and came to the conclusion that the Hawthorne effect was not important in this type of experiment.[3]

STATISTICAL TREATMENT

The null hypothesis was:

There will be no difference in the mean achievement in manipulative ability or understanding in mathematics, as measured by the Cooperative Algebra I Test, between students using the modern textbook Algebra One by Hayden and Finan, and students using the traditional text Mathematics for Canadians by Bowers, Miller and Rourke.

In other words, the mean scores on the post-test were compared using the following hypotheses:

$$H_0 : \mu_1 = \mu_2$$

$$H_i : \mu_1 \neq \mu_2$$

where μ_1 and μ_2 are the population means of the post-test scores from

[2]The Cooperative Algebra I Test is described in a later section of this chapter.

[3]Roland F. Payette, "Reports on Student Achievement in SMSG Courses," Newsletter No. 10, November, 1961 (Stanford: School Mathematics Study Group), pp. 10-11.

which it is hypothesized the experimental and the control groups have been sampled. Test of significance of the difference of means was made at the five per cent level. When the null hypothesis was rejected the direction of the difference in the means was noted. A similar hypothesis was used three times--once for testing the difference of the means of the total scores, and once each for testing the difference in means of the understanding scores and the manipulation scores.

The statistical treatment was simple and straightforward. The control and experimental groups were equal in intelligence according to scores on Laycock Mental Ability Tests. To check the equality, the SCAT quantitative scores of the two groups, written during the grade nine departmental examinations, were compared and found not significantly different for either mean or variance. A t-test was used to check the significance of differences of means and an F test to check the variances.

The two groups were pretested with the Cooperative Algebra I Test the first week of February and at this time there was no significant difference in the mean score of the two groups on this test. Based on the above results the two groups were considered equivalent for the purpose of this experiment. With the two groups equivalent and the same teachers for each group any significant difference in the mean scores on the post-test was attributed to difference in treatment; and since the difference in treatment was due almost wholely to the difference in textbooks, these differences could be attributed to the textbooks.

After the post-tests were given the first week of June, and after it was found that there was no significant difference in variance of

the scores of the two groups for manipulation or understanding or for total scores, the means of the scores for each part of the test and the total scores for the two groups were compared. The significance of the differences in the means in each case was tested by a t-test at the five per cent level. The result of this test will be reported in the next chapter.

A COMPARISON OF THE TWO TEXTS USED IN THIS STUDY

In comparing the two texts an outline of the content covered in the two texts in this investigation will be given and then a discussion of the differences between Algebra One, a modern text, and Mathematics for Canadians,[4] a traditional text, will be given.

The importance of the textbook in determining the content, and to a large degree even the method, of a mathematics program has already been mentioned. Lawrence, in a survey conducted in Los Angeles County, California, found that "the role of the textbook in nearly all districts in the state was reported to be that of providing the major organization and content of the course of study."[5]

Algebra One Content

This book, in both content and approach is quite modern and the difference between it and a traditional text is easily observed from the table of contents for Chapters I to VII which follows:

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[4]H. Bowers, N. Miller, and R. E. Rourke, "Mathematics for Canadians (N.P., J. M. Dent and Sons (Canada) Ltd., and the Macmillan Company of Canada Ltd., 1950).

[5]John Lawrence, "The Application of Criteria to Textbooks in Secondary Schools of Los Angeles County," (unpublished Doctoral Dissertation, University of Southern California, 1961).

A few of the obvious differences can be noted from a casual examination of the table--a more complete discussion will follow. The introduction of new materials such as sets and inequalities is first observed. The use of modern language is apparent in such headings as "Identity Elements," "The Graph of J X J," "Additive Inverse," and "Set R."

Of course, some of the old familiar topics can be noticed in this table such as "Equations," "The Pythagorean Theorem," "Addition of Polynomials," "More Division; Roots," and "The Product of Monomials." However, a more careful examination will show the difference in emphasis even with these topics. The authors report that the text has been used by over fourteen hundred students in twenty-four different schools

in the United States, and that the response of the students and teachers were favorable.

<u>Contents of Mathematics for Canadians</u>

This book has been the authorized text in Alberta since 1953. The Algebraic Content of this book, which was covered by the control group is very similar to other traditional introductory courses for Canadian senior high schools. Because the table of contents is not very descriptive,[6] in this case, a brief outline of the material covered will be given. The first chapter is a review of material covered in junior high school. Topics reviewed are the operations with signed numbers, adding and subtracting like terms, multiplying monomials, reduction of fractions, formulas, equations, and verbal problems. The next chapter deals with solving equations in two unknowns. The graph of the first degree equation is discussed and this is extended to the solution of a linear system of equations in two unknowns. The graphs are used to illustrate that a system of two linear equations in two unknowns may have one solution, no solution, or an unlimited number of solutions. Another chapter is devoted to formulas and graphs; circular, bar, and line graphs are discussed. A chapter is devoted to the solution of quadratic equations by factoring. Factoring is considered as being of six types in the final chapter, but only three were covered by the control group--common factor, trinomial, and difference of squares.

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[6]Only the titles of the chapters are given, which in most cases give little indication of the content of the chapters. For example, "Re-union with Algebra" and "More About Algebra" are the titles of the first and last chapters.

The uses of factoring in working with fractions completes the course.

How Does the Modern Text Differ from the Traditional Text

In this discussion Algebra One by Hayden and Finan will be referred to as the modern text and Mathematics for Canadians by Bowers, Miller and Rourke as the traditional text.

A greater part of the following discussion will describe the modern text, as the content and method of presentation of the traditional test will be familiar to the reader.

It has already been noticed that the modern text contains new topics such as "set", "inequalities", and "absolute value", and that the traditional text does not; also, mention has been made of the difference in language of the two texts. However, this is not the fundamental difference in the two texts. The basic difference is a difference in organization and point of view.

A statement by Beberman applies to the modern text:

> In looking at the content of the new programs for grades 9 - 12, I am impressed more by the attempt to organize the traditional subject matter along logical lines than by the inclusion of new subject matter. What has heretofore been a year spent on learning an assortment of isolated rules and techniques which are quickly forgotten or confused is now a year devoted to learning (preferably discovering) a few basic principles and considering some logical consequences of the principles.[7]

The organization of the subject matter of the modern text is stressed in a way that will enhance the understanding by the student of

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[7]Max Beberman, "The Old Mathematics in the New Curriculum," Educational Leadership, XIX (March, 1962), p. 375.

the structure of mathematics. Basic concepts get more attention in the modern text, whereas more attention was given to mechanical manipulation in the traditional text. The modern text puts much more emphasis on the postulational method of mathematics than does the traditional text which does nothing to make the student aware that the postulational method of mathematics is as important in algebra as it is in geometry. The traditional text does not make clear that postulates are no longer accepted as self-evident truths, but merely acceptable assumptions, as the modern text does. The modern text makes clear that these assumptions are creations of the mathematicians; that it is only after he is convinced of the consistency and validity of his conclusion that he may become interested in the physical applications of the model; that all mathematical systems are creations of man's intellect, its distinctive structure determined only by the basic assumptions and undefined terms; and that some terms must remain undefined to avoid circuity of definition.

"The axiomatic approach though not discussed formally, is inculcated through practice"[8] in the modern text. The student is made aware that a mathematical system or structure (1) contains a set of undefined elements, (2) contains an equal relation, (3) has at least one operation, (4) has a set of postulates or axioms, (5) has theorems or propositions generated from the first four components. This material is not presented formally to the student, but gradually developed by example. The authors refer to this in discussing set theory as a mathematical system,

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[8]Hayden and Finan, op. cit., p. iv.

in their teacher's supplement, "We feel that the teacher should adopt some such approach in discussing mathematical systems so that the student will eventually appreciate the postulational nature of the entire subject."[9]

In Chapter III of the modern text the authors do a commendable job of abstracting the fundamental properties of the set of counting numbers and zero on a level comprehensible to grade ten students. This is followed by the development of the rational numbers, and the real numbers. In each case properties of these numbers are abstracted, and the number system through the reals is carefully developed. As each extension of the number system is made, the student is reminded that these new systems are created by man and that "just as the mathematician is free to make up any sets he pleases, so he is free to define operations any way he pleases."[10] The modern text shows the student that an operation is entirely determined by a definition and that he need only refer to the definition when in doubt. However, each new set of numbers is introduced with some historical background and justification for enlarging the system, and it is pointed out to the student that since some definitions of operations are more useful than others that the mathematician will choose the more useful one. It is clearly presented that each new set of numbers is an extension of the former sets and that these sets are sub-sets of the new set. "When we define division in F, we must take care not to contradict our definition of division in J, since J is a sub-set of F."[11] Since this is the case new definitions

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[9]Dunstan Hayden and E. J. Finan, Teacher's Supplement Algebra One, (Boston: Allyn and Bacon, Inc., 1961), p. 21.

[10]Hayden and Finan, Algebra One, op. cit., p. 152. [11]Ibid.

of operations must not contradict former ones. Also, a justification of the new definitions for operations is based on the desire to have the commutative, associative, and distributive laws--especially the distributive--to apply to the new set. The physical justifications and applications are not dealt with until the abstract mathematical ideas of the set have been developed. The traditional text does not treat the number system.

The modern text communicates a feeling of the unity of its materials to the student better than does the traditional text. One way this is accomplished is by introducing sets in the first chapter and by using it throughout the book in developing subsequent materials. For example, set language and symbolism are used in solving equations and inequalities in graphing. Set theory is used to simplify the concept of function--"Let S be any set. A subset of S x S is called a function in S x S if and only if no two ordered pairs in it have the same first elements"[12]--to simplify the concepts of range and domain of a function. Another example is the use of the idea of intersection of sets to clarify the solution of systems of linear equations and inequalities.

A second way a sense of unity is achieved is by presenting the commutative, associative, and the distributive laws; and the inverse and identity elements for addition and multiplication early in the book and restating them for each extension of the number system, and using them--particularly the distributive law--in justifying operations with

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[12]Ibid., p. 58.

negative numbers, fractions, radicals, and polynomials.

Thirdly, each chapter is made to be a logical extension of those that precede it. For example, the addition of the integers is defined in terms of addition and subtraction of the natural numbers. The traditional text does not succeed to the same extent of conveying the feeling of the unity for its mathematics.

Some sound pedagogical procedures are made use of in the modern text as well as sound mathematics procedures. One such procedure is the encouraging of students to discover mathematical ideas for himself. Throughout the explanatory sections of the book, questions are directed towards the student. The student is encouraged to follow the thinking of the author as shown by:

> Under what conditions will two counting numbers have a counting number as their quotient? The student should ponder these questions and try to find some answers. The incomplete sentences below may suggest some trains of thought:. . .[13]

Following this is listed five incomplete statements that lead the student to the desired definition. The authors also make considerable use of discovery-type exercises intended to lead the student to a deeper understanding of mathematical concepts. The exercises often prepare the student for the next section in the text by leading him to anticipate what is to follow. The "Special Project", at the end of each chapter is useful in encouraging the student to think for himself and to expand ideas beyond the level presented in the text.

The traditional text also makes some use of the discovery method as illustrated by the following two quotations:

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[13]Ibid., p. 73.

> In drawing graphs, as you recall from Chapter II, we make use of pairs of numbers. How can we build up a table of number-pairs from a formula? Your previous work gives you the answer. . .
>
> Three types of graphs in common use will now be illustrated. As you look at them decide what information each graph attempts to portray.[14]

However, the traditional text does not use the discovery method as extensively as does the modern text. The asking of students to develop some theorems is also a type of discovery exercise found in the modern text, which gives the student practice in the deductive method. Although the modern text does not give special space to the discussion of the postulational method and logical developments of proofs the concept appears repeatedly in definitions and arguments throughout the text. In the Handbook the authors say:

> If one attempts to discuss modern mathematics he can hardly avoid the notion of postulational thinking even though he may use other words for it. . . .We urge the teacher to reflect on this notion and the vocabulary that goes with it so that it gets emphasis or display as one goes through the Text. The authors feel that this concept is one that students can acquire without the expenditure of extra time.
>
> . . . .
>
> The reader has already noticed the extensive use of *if*, *then*, *if and only if*, *necessary*, *sufficient*, *and necessary and sufficient*. Again we urge the teacher to devise ways to use these *logical connectives* correctly in class conversations. Since one never has enough time to get everything done it is hoped a way will be found to get some of the students to understand these three concepts in the ordinary class work and that it will not be necessary to set aside certain periods for this purpose.[15]

The traditional text does not present the postulational method in its discussion of algebra.

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[14]Bowers, Miller and Rourke, *op. cit.*, pp. 50 and 54.

[15]Hayden and Finan, *Teachers' Supplement*, *Algebra One*, *op. cit.*, p. 60.

Another pedagogically sound procedure, found more in the modern text, is the providing of negative illustrations to help the student understand a concept and to see the need for the concept. To illustrate, when discussing the commutative law, examples are given of systems that are not commutative. In the same way, examples that are not functions are given to help clarify the concept of function.[16] Negative illustrations of closure are discussed where applicable.

An effort is made in the modern text to present its material soundly, so that sutdents will not have to unlearn ideas if they should pursue their study to higher mathematics. Of course, it is not practical to present algebra at the grade ten level so it will be completely rigorous. An instance is the fact that no definition is given for real numbers. The authors feel that all the good definitions of real numbers are too difficult for students at this level and rather than give one that would have to be discarded later, they omitted it altogether.[17] Mathematicians try to minimize the number of assumptions in a mathematical structure, but pedagogically this may not be desirable. It is still mathematically sound to assume more than the absolute minimum and this is often sounder pedagogically for immature students. In other words, independence of postulates is relatively unimportant as long as they are consistent. For this reason the authors of the modern text state: "The set of thirteen postulates given in this chapter need not be considered the best set and certainly not the only set possible. Our object was to compose a useful set which would serve throughout high

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[16] Hayden and Finan, Algebra One, op. cit., p. 58.

[17] Hayden and Finan, Teachers' Supplement, Algebra One, op. cit., p. 42.

school."[18]

Another illustration of the modern text laying a good foundation is the development of a concept of an integral domain, which will be useful to students studying modern algebra later on, although the name 'integral domain' is not mentioned or formally discussed. The same is true of additive and multiplicative groups.

The authors are quite honest in telling students that in many cases they are now making assumptions which they will prove in later courses. Of course, one of the complaints of university lecturers for years has been that the traditional texts did not prepare students for the university courses. It is hoped that the modern texts will do better in this respect.

Henderson, in a review of the modern text, observes:

> The most striking feature is the accuracy and conciseness with which the authors have tried to state the definitions and theorems. They have stated them in mathematical terms and have not resorted to inaccuracies for the sake of ease of understanding.[19]

Although the present writer would not agree that this is "the most striking feature," he will agree it is one of the text's strengths.

Mention was made previously of the differences in language, terminology, and symbolism. Two examples from the modern text will be given to illustrate the mathematical precision:

> Definition 2-1
>
> The solution set for an equation containing variables or an inequality containing variables is the set of all replacements for the variables which make the equation or inequality a true statement.[20]

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[18]*Ibid*., p. 60.

[19]Clarence H. Barthelman, "Books," *The Mathematical Teacher*, LV (May, 1962), p. 397.

[20]Hayden and Finan, *op. cit*., p. 46.

Definition 3-1

For any set S, if it is always possible to perform a certain operation on any members of S so that the answer is a member of S, then set S is said to be closed under that operation.[21]

Some other differences in terminology found in the new text are:

The modern text distinguishes between number and numeral; the traditional does not.

The modern text calls a statement of equality containing a variable an open sentence.

The traditional text speaks of roots of an equation, the modern of solution sets of equations.

The modern text talks of ordered pairs, ordered triples, etc., in discussing the solution sets of sentences of several variables; the traditional does not.

To conclude, a few of the minor characteristics of the modern text that sets it apart from the traditional text will be listed:

Extensive use of the number line in introducing the number sets.

The domain of the variables is conscientiously specified.

Addition and multiplication are given as basic operations; subtraction and division are simple inverse operations.

Order relations are discussed.

The idea of one to one correspondence is introduced.

A variable is explained as a placeholder.

The solution of equations and inequalities is made by placing stress on the importance of moving from one equivalent equation to

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[20]Hayden and Finan, _op_. _cit_., p. 46.

[21]_Ibid_., p. 70.

another until the final solution is reached. Equivalent equations are defined as two equations having the same solution set.

A brief introduction to probability is given using set ideas.

Many of the exercises in the modern text call for original and creative thinking on the part of the student, not just manipulation.

The authors claim that all of the traditional topics have been retained and that there are sufficient exercises for the development of manipulative skills. However, there are fewer manipulative exercises in the modern text than in the traditional.

RECAPITULATION

The differences in the two texts are in content, language, and emphasis. The main difference is the difference in emphasis. The emphasis in the modern text is on understanding of structure while the traditional text emphasizes manipulation to a greater degree. The modern text gives more attention to recurring patterns in mathematics; it gives careful attention to developing the structure as a deductive-axiomatic system. The traditional text discusses the many properties of the models, whereas the modern text studies the abstract structure of the mathematics system and then applies it to the many physical models. In this way the modern text makes it necessary for the student to study only the properties and rules of the mathematical system and then apply these common rules and properties to the individual physical models. The modern text has an over-all unity because of its emphasis on structure. Many minor differences were discussed.

THE TESTS INVOLVED IN THE STUDY

The school records were used to obtain the Laycock Mental Ability scores and the SCAT: Quantitative scores. The "new" Cooperative Algebra I Test Form A was given on February 7th and 8th and then again as a post-test the first week of June 1963. The I.Q. tests were given in October 1962.

The new Cooperative Algebra I Test was chosen because it contained a larger proportion of questions that tests for more than mere manipulative skills, has more non-routine problems, and because of its inclusion of more questions based on topics that are receiving greater emphases in the modern programs. The publisher, Educational Testing Service, comments as follows on the new Cooperative Tests in Mathematics:

> Everyone associated with schools is aware of the ferment in mathematics education.
> The Co-operative Test Division has followed developments in this field with great interest and has taken major trends into consideration in planning new tests and revising existing instruments.
> New Tests in Arithmetic, two levels of Algebra, and Geometry were published in 1962 for use at the junior and senior high school level.
> Some of the newer emphases in language and content can be found in these tests, but in the main, important aspects of traditional mathematics continue to be measured. Students in both traditional and new courses will find the test stimulating and understandable.
> When possible, major shifts in mathematics curriculum have been represented. Inequalities can be found throughout the series of the tests. On the other hand, formal algebra of sets is not included, although the underlying notion of the sets is formally present.
> Content Measures students' understanding of mathematical concepts in each subject area and his ability to reason with insight.[22]

Mention is also made of these tests in the Educational Testing

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[22] Tests Materials and Services, Cooperative Tests Division, Educational Testing Service, Los Angeles 27. pp. 22-23.

Services Annual Report, 1961-1962. "These tests measure outcomes desired from both traditional and more modern mathematics courses."[23]

As the publishers noted, the tests are suitable for both modern and traditional courses; this made the tests especially suitable for this study. In this way the present investigation is an improvement over some of the previous ones, as all the previous studies that used standardized tests used tests that were specifically prepared for the traditional programs. The impression must not be left that these tests were unfair to the traditional student. There were no questions on the test that could be answered only by the modern program student and not by the traditional. But the test was a better measure of the outcomes, such as understanding, desired of the modern program than were other standardized tests. Besides the emphasis on understanding, there is more emphasis on graphs and inequalities. A copy of the test can be found in the appendix.

The publisher's report, in a leaflet accompanying the test, that the reliability, as computed by the Kuder-Richardson Formula 20 on 964 students in thirteen suburban schools in six states, was .85.

It was decided to use the test as a measure of both manipulative skill and meaningful interpretation. The chairman of the thesis committee, the investigator and one other graduate student divided the questions on the test into two groups, one group consisting of questions requiring only mechanical manipulative skill and the other group consisting of those requiring a meaningful interpretation or an indication

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[23] Annual Report 1961-1962, Educational Testing Service, Los Angeles 27, p. 71.

of understanding on the part of the student. Each person made a division of the questions without knowledge of the way the other two had divided them. There was agreement among the three for thirty-five out of forty questions. In the five cases of disagreement the questions were placed according to majority decision. Of the forty questions on the Test, twenty-one questions were considered to measure only manipulative skill and nineteen to require some interpretation and understanding. By dividing the test into two parts in this manner, a total score was obtained, a score as a measure of manipulative ability, and a score as a measure of understanding. These scores will be referred to as manipulation, understanding, and total scores. These three scores are reported individually in the next chapter.

SUMMARY

This chapter outlined the origin of the Edmonton Public School System's Experimental Course in Grade Ten Algebra, and the relation of this investigation to that program. In the next two sections the setting up of the experiment and the statistical procedures were discussed. It was shown that the experimental and the control group were groups consisting of thirty-five students each, equated on the basis of ability and achievement on the pre-test. A t-test was used to test the significance of the difference of means on the post-test. A comparison of the two texts used in the project was presented next, illustrating the difference in content, language, and emphasis; the latter was described as the main difference. And finally, the advantages of the <u>Cooperative Mathematics Test Algebra I</u> was presented, emphasizing the

fact that the test was equally suitable for testing the desired outcomes of both traditional and modern courses.

CHAPTER IV

RESULTS OF ANALYSIS

This chapter will report the results of the testing and statistical treatment described in the last chapter.

The students will be designated by numbers from one to thirty-five in each of the two groups. The same number will be used to refer to a particular student throughout the chapter.

TableIII, in the appendix, gives the I.Q., based on the Laycock Mental Ability Test, and the SCAT Quantitative raw scores of students in both the control and experimental groups. The mean I.Q. of the two groups was 117, the same for both groups, to three significant digits. The range of I.Q. was 98 to 142 for the control group, and 91 to 141 for the experimental group. The means of the SCAT Quantitative scores are slightly different in the third digit but far from being significantly different. The range of SCAT scores for the control group was 28 to 49, and for the experimental group the range was 22 to 49.

Table IV, in the appendix, shows the results of the pre-test. As described previously, the Cooperative Mathematics Test Algebra I was divided into two parts. One part to test understanding and meaningful interpretations and the other to test for mechanical manipulative skill. Table IV gives the scores on these two separate parts and then the total score for the control and the experimental group. The mean of the score on understanding for the control group was 9.63 to three significant digits and that of the experimental group was 8.80. These

two means were tested with a t-test[1] and were found not to be significantly different at the five per cent level.

The means of the scores on the manipulative part of the test are 12.2 for the control group and 12.6 for the experimental group. Again a t-test showed that the difference was not significant at the five per cent level. The means of the total scores are 21.8 and 21.4 for the control and the experimental groups respectively. This is not significantly different for the two groups at the five per cent level. The significance of the difference of variances between the scores of the control group and the experimental groups for I.Q., SCAT, and the three parts of the Cooperative pre-test were tested using an F test. In each of the five cases the difference was found to be insignificant at the five per cent level.

The mean and the standard deviation of the pre-test and the post-test scores for the three parts of the Cooperative Test for both the control and the experimental groups are given in Table I, page 76. The scores of all students on the post-test are given in Table V in the appendix. The means are shown below for easy comparison.

	Understanding	Manipulation	Total
Control group	10.5	14.6	25.0
Experimental group	12.0	15.7	27.7
Difference	1.5	1.1	2.7

As was stated earlier, it was assumed that the distribution of

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[1]George A. Ferguson, Statistical Analysis in Psychology and Education (New York: McGraw-Hill Book Company, Inc., 1959), pp. 136-140.

TABLE I

MEAN AND STANDARD DEVIATION OF SCORES ON THE COOPERATIVE MATHEMATICS ALGEBRA I TEST

	Control Under-standing	Control Manipula-tion	Control Total	Experimental Under-standing	Experimental Manipula-tion	Experimental Total
			PRE-TEST			
Mean	9.63	12.2	2.18	8.80	12.6	21.4
Standard Deviation	2.16	2.72	3.64	2.69	2.70	4.55
			POST-TEST			
Mean	10.5	14.6	25.0	12.0	15.7	27.7
Standard Deviation	2.68	2.77	4.46	3.49	2.28	5.23

this variable in the population was normal. Winer says in this respect:

> The Work of Box (1954) also indicates that the t-test is robust with respect to the assumption of normality of the distributions within the treatment populations. That the type 1 error of the decision rule is not seriously affected when the population distribution deviates from normality. Even when population distributions are markedly skewed, the sampling distribution of the t statistic, which assumes normality, provides a good approximation to the exact sampling distribution which takes into account the skewness. In summary, preliminary tests on the structural model for the t-test which assumes homogeneity of variance and normality of distributions are not of primary importance with respect to type 1 error, particulary tests for normality of distribution.[2]

Table II gives the means and the standard deviation of the scores for the I.Q. and SCAT test for the control and experimental groups.

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[2]B. J. Winer, Statistical Principles in Experimental Design (New York: McGraw-Hill Book Company, Inc., 1962), pp. 33-34.

TABLE II

I.Q. AND SCAT

	Control		Experimental	
	I.Q.	SCAT	I.Q.	SCAT
Mean	117	38.5	117	38.9
Standard Deviation	9.68	5.16	12.58	6.9

Considering the above and also that there is no reason to believe that the scores on this test in the population of grade ten students would not be normally distributed, the assumption of a normal distribution in the population should not adversely affect the conclusions of this experiment.

The homogeneity of variance was checked by the use of an F test for the total score, and also for the understanding and manipulation scores. In each case the difference in variances between the control and the experimental group was not significant at the five per cent level. (In fact, they were not significant even at the ten per cent level.)

A t-test was applied to test the significance of the difference between the means of the total scores on the post-test. There was a significant difference between the two means, at the five per cent level. That is to say, there would be only five chances in one hundred of a difference as large as this occurring when the population means were equal. On this basis, the null hypothesis, concerning the total score, was rejected. There was a difference of 2.7 in the means favoring the experimental group. One might say that the group using the modern text

did significantly better than the group using the traditional text on the post-test as a whole; the maximum probability of error of such a directional statement is $\frac{\alpha}{2}$.[3] A t-test was also applied to test the significance in the difference in means between the understanding scores of the experimental and the control group. Again the difference was significant at the five per cent level. The null hypothesis for the understanding score was rejected. The difference of 1.5 again favored the experimental group. This might be interpreted as meaning that the group using the modern text did significantly better than the group using the traditional text in the section of the post-test based on understanding, with the maximum probability of error being $\frac{\alpha}{2}$ as before.

There was a difference of 1.1 in the means of the manipulative score. A t-test was applied to test the significance of this difference between the experimental and control groups, but in this case the difference was not significant, at the five per cent level. In other words, there were no grounds for rejecting the null hypothesis in this case. There was no significant difference between the manipulative abilities of the group using the traditional text and the group using the modern text as measured by the manipulative section of the post-test.

There existed a possibility that there was no significant difference between the post-test and the pre-test scores for understanding, manipulation, or total for either the control or the experimental groups. To check this possibility, a t-test to test for significance of the difference between two means for correlated samples was used.[3]

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[3]H. F. Kiaser, "Directional Statistical Decisions," Psychological Review, LXVII:3 (1960), pp. 160-167.

[4]Ferguson, op. cit., pp. 138-140.

This test was used six times--first to test the difference between the post-test and pre-test scores of each of understanding, manipulation, and total for the control group; then to test the differences between the post-test and pre-test for understanding, manipulation and total scores for the experimental group. In each case there was a significant difference at the five per cent level. From this one might conclude--with the same reservations as stated on page 78--that there was a measurable improvement in understanding, manipulation, and total ability for both the control and the experimental groups.

It might be noted that the above gains in scores on the test during the period of the experiment were highly significant being significant at the .1 per cent level for all except the gain for the control group in understanding.

Figure 1, in the appendix, is presented to illustrate the relative gains made by the control and experimental groups during the period of the experiment.

It was felt by the investigator that there was a possibility that the variability of the scores might be increased by the use of the modern text because the lower ability students might find the text too difficult. However, a check on the significance of the difference of the variance, using an F test, between the pre-test scores and the post-test scores of understanding, manipulation, and total were made and in none of the three cases was the difference significant at the five per cent level. An F test was used, also, to check the difference of variance between each of the three scores of the pre-test and the post-test for the control group and were found to be insignificant at the

five per cent level.

In conclusion the differences were not significant at the beginning of the experiment between the experimental group and the control group in I.Q. scores, SCAT quantitative scores, scores on the Cooperative Mathematics Test Algebra I, or scores on either section of this test; this applies in both the cases of variance and means at the five per cent level. At the end of the experimental period it was found that both groups had made measurable gains (significant at the five per cent level) in all parts of the cooperative test and that there was a significant difference in the means of the total scores on the Cooperative Mathematics Test Algebra I and a significant difference in the means of the scores on the understanding section of this test. In both cases the difference favored the group using the modern text, Algebra I by Hayden and Finan over the group using the traditional text Mathematics For Canadians, by Bowers, Miller and Rourke. The difference was attributed to the difference in treatment due to the use of the different texts. The difference in means on the manipulation section was not significant, and it would appear that the use of the modern text improves understanding, but not manipulative skill. Neither the use of the traditional text or the modern text affected the variability of the scores.

CHAPTER V

SUMMARY, CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

This chapter will summarize the study, give the conclusions drawn from the investigation, outline the limitations of the study and state some implications for further research.

PURPOSE OF THE STUDY

This thesis has illustrated the considerable lack of agreement among authorities, and the indecisiveness of research. The writer submits that it is important to admit that this is the situation at present and not to pretend that the modern approach has universal sanction. This is not to say that research and professed convictions of educational leaders do not offer guidance, but rather that it makes clear, in the final analysis, that many curriculum decisions must be based on value judgment arrived at only after carefully weighing all of the evidence. It is the hope of the investigator that this study will provide one more useful piece of evidence upon which to base such decisions.

The Mathematics Subcommittee of the Senior High School Curriculum Committee is facing the decision of when and if to introduce a modern mathematics program in the senior high schools of Alberta; again the investigator hopes that this study will provide some evidence that will aid in this decision.

EXPERIMENTAL DESIGN

The control group of thirty-five grade ten students, using the traditional textbook, Mathematics for Canadians by Bowers, Miller and Rourke, had a mean I.Q. not significantly different from the experimental group of thirty-five grade ten students using the modern text, Algebra One, by Hayden and Finan. This equality of ability was verified by comparing the SCAT quantitative scores from the grade nine departmental examination and the scores on a pre-test given the first week of February. Two teachers were involved in the experiment, in two different Edmonton schools. Each teacher taught one control and one experimental class. A post-test was given in June, the Cooperative Mathematics Test Algebra I, divided into parts so as to give a measure of understanding (nineteen questions) and a measure of manipulative skill (twenty-one questions). The means and the variances of the scores on these two parts and also the means of the total scores for the control and the experimental groups were found. When the significance of the difference in variances between the two groups was found to be insignificant at the five per cent level for the two groups, a t-test was used to test the significance of the difference in means between the control and experimental groups for the three post-test scores.

SUMMARY OF RESULTS AND CONCLUSIONS

The means of the scores were found to be significantly different at the five per cent level for the total test and for the part testing understanding but not for the part testing manipulative skill. From this

it can be concluded that the use of a modern text book such as Algebra One by Hayden and Finan can significantly improve the understanding of algebra, as evidenced by the greater ability to solve non-routine problems, by senior high school students. On the other hand, the fact that there was no significant difference in manipulative skills would indicate that the students using a modern text do at least as well in this regard. That is, there is no loss in manipulative skills resulting from the greater emphasis on understanding the structure of algebra and the lessened emphasis on manipulation found in the modern text. It can be concluded then, that there is an advantage in using a modern text as compared to a traditional text, at the grade ten level at least, and with certain teachers, if understanding is one of the objectives of the algebra course. This conclusion, that students have a better understanding of algebra when using a modern text was to be expected as the modern text puts greater emphasis on the understanding of mathematics through its emphasis on structure. The conclusion is much the same as that reached by two other similar investigations described earlier. It will be recalled that Kushta found no significant difference between a control group using a traditional text and an experimental group using materials organized around certain unifying concepts, in manipulative skills, but that there was a significant difference in favor of the experimental group in understanding of the nature of mathematics.[1]

In another study reported earlier, Loman found in comparing

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[1] A fuller discussion can be found on page 38 of this thesis.

students using UICSM with those using a traditional text that there was a significant difference in understanding of basic mathematical concepts in favor of the group using the UICSM materials only at the upper one-third intelligence level. Again there was no significant difference in manipulative skills.[2] Thus the findings in the cases of Kushta and of Loman agree to a large degree with the findings of the present study.

It was pointed out in chapter two of this thesis, that in many cases where there was found no significant difference in achievement of students following a modern program and one following a traditional program, the investigator used standardized tests that were developed to measure the achievement towards attainment of outcomes desired of the traditional courses. It was also shown that the <u>Cooperative Mathematics Test Algebra I</u> used in this investigation was designed to measure outcomes desired of both traditional and modern courses. The writer believes that it was the difference between the measuring instrument used in the present study and the instruments of those investigators using older standardized tests, that lead to the present investigator's finding a significant difference while the others found no such a difference in achievement. In any case, the conclusions of this thesis is supported by at least two other studies--Kushta's and Loman's.

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[2] A fuller discussion of this experiment can be found on page 39 of this thesis.

LIMITATIONS OF THE STUDY, RECOMMENDATIONS AND IMPLICATIONS FOR FURTHER RESEARCH

This study did not attempt to determine whether it was the difference in content, difference in language, or difference in emphasis in the two texts that produced the difference in achievement. It is conceivable that any one of these or any combination of them could have been responsible for the difference in achievement. There is much research needed to determine this. Then the differences in content could be further broken down into individual factors to determine which, if any, are effective in producing changes in achievement. The same could be done with language differences and emphasis differences. There is a need for information of the efficiency of the pedagogical difference as well as the mathematical difference.

The experimental group and the control group were not equal as to number of boys and girls. Several studies have shown that sex is not an important factor in determining mathematical achievement. For example, Alexander showed that sex was not related to arithmetic reasoning.[3] In a well-developed, statewide study in Minnesota it was found that there was no statistically significant difference between the mean final achievement in mathematics between boys and girls.[4]

Other factors that were not considered in matching the

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[3]Vincent Eugene Alexander, "The Relations of Selected Factors to the Ability to Solve Problems in Arithmetic" (unpublished Doctoral dissertation, University of Southern California, Los Angeles, 1959).

[4]Analysis of Research in the Teaching of Mathematics, 1957 and 1958 (Washington: The U.S. Office of Education, 1959), p. 14 and p. 38.

experimental and control group were chronological age and socio-economic status. The former was not considered important and no information was available on the latter. Alexander, also, showed that chronological age and socio-economic status were not related to arithmetic reasoning. It is unlikely that they would be related to algebraic reasoning.

The research of this study was limited to the two groups of thirty-five students whose mean I.Q. was 117. A useful project would be to repeat the experiment with groups of lesser ability.

Many of the outcomes expected of the modern mathematics courses are long-term objectives. One such outcome hoped for is better grades in later mathematics courses--especially university courses. A long-term follow up project would be necessary to determine whether this expectation would be realized. Another expectation of the 'Moderns' is that there will be greater transfer to the solution of life's problems in the future. To determine whether this expectation is justified would require a complicated follow up project. A similar short-term study might determine if a modern program led to greater transfer in school work.

Most research, including that reported in this thesis, has been concerned with the testing of acquisition of skills, knowledge, and understanding in the modern program. Since it is claimed by the proponents of modern mathematics, that the attitudes towards mathematics, such as a liking for mathematics, appreciation of the importance of mathematics, and a desire to work with mathematics (motivation), are improved by giving the student a modern course, there is much room for research in this area as well.

Much is said and written today about creativity in all school

subjects. What is the effect on creativity of the modern mathematics programs? As soon as proper measuring instruments are available, this question will need to be answered by research.

The modern algebra programs put considerable emphasis on postulational reasoning. Will the new programs improve the ability of students to do this type of reasoning? Will they improve the ability to generalize relationships?

Finally, the field is wide open to the researcher to investigate the relations between modern mathematics programs and team teaching, programmed learning, and audio-visual aids.

There are many unanswered questions about the modern programs, but the curriculum makers cannot wait until all the answers are in, they must make decisions now. The investigator recommends that there be a gradual introduction of the modern programs on an experimental basis in schools of Alberta, as teachers become qualified to handle the new programs. This study would indicate that by following such a plan, not only can gains be expected in understanding but there will be no harm done in the area in which the traditional program puts most of its emphasis, the area of manipulative skills.

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APPENDIX

TABLE III

THE I.Q. AND SCAT QUANTITATIVE RAW SCORE OF STUDENTS IN THE CONTROL AND EXPERIMENTAL GROUPS

Student's Number	Control I.Q.	Control SCAT Q	Experimental I.Q.	Experimental SCAT Q
1	119	30	112	41
2	117	43	133	46
3	107	40	108	36
4	109	35	105	31
5	123	45	121	48
6	128	33	131	45
7	116	38	117	45
8	107	33	97	40
9	104	43	141	40
10	134	28	125	40
11	109	31	132	38
12	105	30	120	48
13	122	38	125	43
14	110	41	107	37
15	115	39	97	43
16	118	49	104	39
17	115	42	122	44
18	109	46	103	27
19	131	43	116	48
20	126	41	115	49
21	120	47	127	47
22	116	34	91	22
23	116	32	131	38
24	116	37	129	41
25	133	35	111	31
26	125	43	132	29
27	102	36	103	25
28	98	38	115	39
29	128	42	96	30
30	114	38	132	36
31	113	41	103	34
32	115	40	122	46
33	142	42	125	34
34	111	42	122	38
35	123	33	123	42
Mean	117	38.5	117	38.9
Standard Deviation	9.68	5.16	12.58	6.9

TABLE IV

SCORES FOR THE CONTROL GROUP AND THE EXPERIMENTAL GROUP ON THE PRE-TEST

Student's Number	Control Under-standing	Control Manipu-lation	Control Total	Experimental Under-standing	Experimental Manipu-lation	Experimental Total
1	9	9	18	11	14	25
2	9	11	20	11	14	25
3	15	8	23	5	9	14
4	7	12	19	4	8	12
5	10	13	23	13	14	27
6	11	12	23	7	14	21
7	7	14	21	7	15	22
8	8	12	20	7	12	19
9	10	16	26	12	11	23
10	11	15	26	13	15	28
11	6	13	19	7	15	22
12	9	9	18	12	13	25
13	11	14	25	6	11	17
14	12	12	24	11	9	20
15	7	11	18	11	10	21
16	12	17	29	10	9	19
17	9	14	23	8	11	19
18	10	10	20	6	10	16
19	13	14	27	10	17	27
20	12	16	28	13	16	29
21	12	11	23	13	13	26
22	11	4	15	2	13	15
23	9	9	18	8	18	26
24	4	10	14	11	17	28
25	11	13	24	8	11	19
26	10	16	26	9	8	17
27	10	13	23	5	9	14
28	11	12	23	7	13	20
29	7	12	19	7	14	21
30	11	16	27	7	14	21
31	7	12	19	5	12	17
32	9	8	17	13	16	29
33	10	14	24	10	14	24
34	9	11	20	10	13	23
35	8	14	22	9	9	18
Mean	9.63	12.2	21.8	8.80	12.6	21.4
Standard Deviation	2.17	2.72	3.64	2.89	2.70	4.55

TABLE V

SCORES FOR THE CONTROL GROUP AND THE EXPERIMENTAL GROUP ON THE POST-TEST

	Control			Experimental		
Student's Number	Under-standing	Manipu-lation	Total	Under-standing	Manipu-lation	Total
1	11	15	26	8	19	27
2	11	12	23	15	17	32
3	10	17	27	11	14	25
4	9	16	25	6	14	20
5	13	17	30	15	16	31
6	12	18	30	11	16	27
7	10	19	29	12	17	29
8	9	12	21	15	14	29
9	14	19	33	14	17	31
10	15	12	27	10	14	24
11	9	13	22	13	16	29
12	7	11	18	15	15	30
13	10	13	23	14	17	31
14	11	13	24	12	16	28
15	13	16	29	13	17	30
16	14	17	31	13	15	28
17	12	13	25	14	18	32
18	13	13	26	4	11	15
19	10	17	27	18	19	37
20	11	16	27	17	17	34
21	15	17	32	15	20	35
22	7	12	19	5	9	14
23	7	9	16	14	17	31
24	12	10	22	16	18	34
25	10	17	27	9	15	24
26	9	18	27	11	14	25
27	13	12	25	5	14	19
28	9	11	20	11	16	27
29	3	15	18	12	16	28
30	9	14	23	15	12	27
31	8	16	24	9	15	24
32	5	10	15	15	19	34
33	13	17	30	13	15	28
34	12	18	30	13	16	29
35	10	15	25	7	13	20
Mean	10.5	14.6	25.0	12.0	15.7	27.7
Standard Deviation	2.68	2.77	4.46	3.49	2.28	5.23

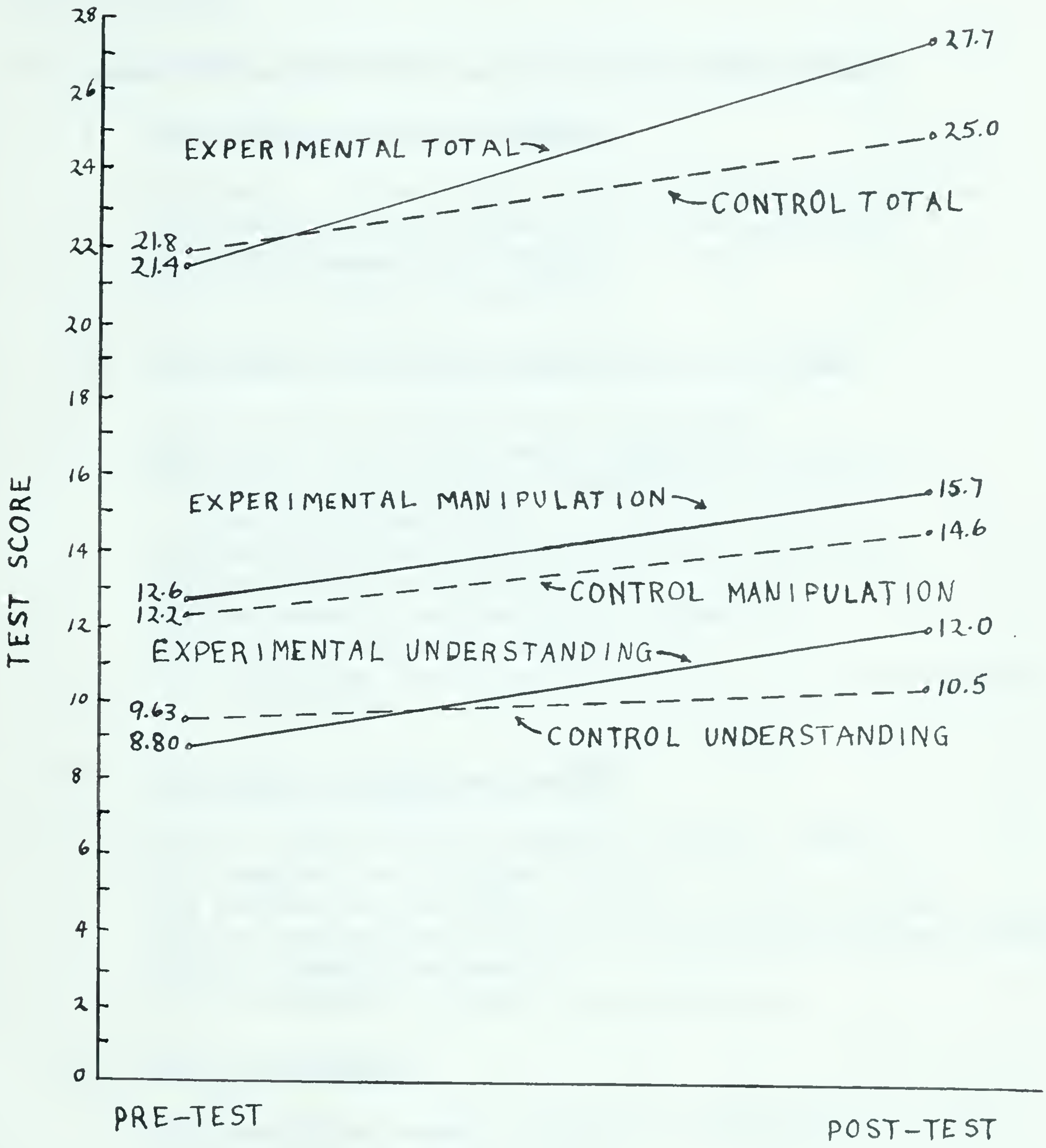


FIGURE 1

A COMPARISON OF THE RELATIVE GAINS IN SCORES OF THE EXPERIMENTAL AND CONTROL GROUPS DURING THE PERIOD OF THE EXPERIMENT FOR THE THREE PARTS OF THE TEST

PROPOSED EXPERIMENTAL COURSE IN GRADE X ALGEBRA

EDMONTON PUBLIC SCHOOL SYSTEM

Course Outline

A. Definitional development of the real number system.

1. The system of natural numbers

(a) Sets - infinite and finite, equivalent, Cartesian
(b) Operations and properties of the operations
(c) Prime numbers and composite numbers
(d) Factorization properties

2. The system of rational numbers of Arithmetic

(a) Fractions and equivalent fractions
(b) Relatively prime numbers and basic fractions
(c) Ordering the rational numbers of arithmetic and the number line
(d) Operations and properties of the operations
(e) The identity elements
(f) Reciprocals and reciprocal properties
(g) Decimal fractions numerals
(h) Conditions involving rate pairs (including equivalent conditions)

3. The system of rational numbers

(a) The positive and negative rational numbers
(b) Ordering the rational numbers
(c) Addition and multiplication of rational numbers
(d) The commutative, associative and distributive properties
(e) The identity elements
(f) The additive inverse and reciprocals

4. The real numbers

(a) Irrational numbers (infinite decimals)
(b) Squares and square roots
(c) Algebraic properties of the real numbers, including the well-defined properties of addition and multiplication
(d) Other properties - order, completeness, density
(e) Solution sets of conditions for equality and inequality
(f) Co-ordinate axes and the real plane
(g) Conditions in two and three variables
(h) Direct variation, including $y = kx$, $y = kx^2$, $y = \frac{k}{x}$, etc.

B. Axiomatic approach to the real numbers

1. The real numbers as a field under addition and multiplication

   (Algebraic properties, subtraction and division, geometrical representation of real numbers).

2. The real numbers as an ordered field

   (Inequalities - order properties of the real numbers)

C. Conditions over the real numbers

1. Conditions for equality (linear and quadratic)
2. Conditions for inequality
3. Systems of linear conditions

D. Mathematical systems (further examples)

1. The algebra of sets and set algebra related to number algebra
2. The group as a mathematical system and elementary group theory.

- - - - - - - - -

N.B. Problem solving throughout the program related to the mathematics under study.

FORM A

ALGEBRA I

COOPERATIVE TEST DIVISION,
EDUCATIONAL TESTING SERVICE,
PRINCETON, N.J. • LOS ANGELES 27, CALIF.

GENERAL DIRECTIONS

This is a 40-minute test. Do not spend too much time on any one question. If a question seems to be too difficult, make the most careful guess you can, rather than waste time over it. Do not worry if you do not finish the test. Your score is the number of correct answers you mark.

Use scratch paper to work problems. Do not make any marks in your test booklet.

Mark all answers on the separate answer sheet. Make your answer marks heavy and black. Mark only one answer for each question. If you make a mistake or wish to change an answer, be sure to erase your first choice completely.

Note how the answer to the EXAMPLE below is marked on your answer sheet.

EXAMPLE

If $2x = 4$, $x =$ (?)

A $\frac{1}{8}$

B $\frac{1}{2}$

C 2

D 4

E 8

Do not turn this page until you are told to.

COOPERATIVE MATHEMATICS TESTS

Algebra I | 40 minutes

1 If $2x + 1 = 7$, then $x = (?)$

A $\frac{1}{4}$

B $\frac{1}{3}$

C 3

D 4

E 11

2 The statement, "A certain number f increased by twice another number n is equal to 30," can be written

F $f + 2n = 30$
G $f + 2f = 30$
H $2f + n = 30$
J $2f + 2n = 30$
K $2nf = 30$

3 $(-5) - (-9) = (?)$

A -14
B -4
C 4
D 14
E 45

4 If $x = y = z = 1$, then $\frac{x - y}{x + z} = (?)$

F -2

G -1

H 0

J $\frac{1}{2}$

K 1

5 $-2x + 5x - 9x = (?)$

A $-16x$
B $-14x$
C $-11x$
D $-6x$
E $-2x$

6 If n is an even number, what is the next larger even number?

F $n - 2$
G $n - 1$
H $n + 1$
J $n + 2$
K $2n$

7 What is the coefficient of y in the expression $2y^5 + 6y^4 - 4y^2 - 5y + 1$?

A -5
B -1
C 1
D 2
E 5

8 If $A = LW$ and if $A = 12$ and $L = 3$, then $W = (?)$

F $\frac{3}{4}$

G 3

H 4

J 12

K 36

9 $\frac{x^7}{x^3} = (?)$

A $x^{3.5}$
B x^4
C x^{10}
D x^{21}
E 3.5

10 Which of the following is equivalent to $x(x + a) - a(x - a)$?

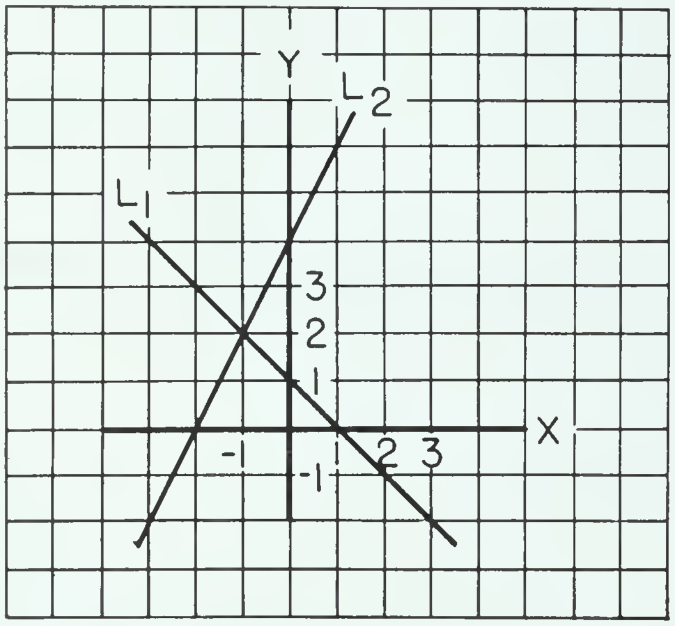
F $(x + a)(x - a)^2$
G $(x + a)^2(x - a)$
H $(x + a)^3$
J $(x + a)^2$
K $x^2 + a^2$

Go on to the next page.

11 If $\begin{matrix} x + y = 4 \\ x - y = 2 \end{matrix}$, then $x = (?)$

A 0
B 1
C 2
D 3
E 6

12



The figure above shows the graphs of two linear equations. What is the solution of these equations?

F $(-2, 4)$
G $(-1, 2)$
H $(-2, 1)$
J $(1, 4)$
K $(1, 1)$

13 If $9x - 63 = 18$, $x = (?)$

A -9
B -5
C 0
D 5
E 9

14 What is the square root of $16b^8$?

F $2b^2$
G $4b^2$
H $4b^4$
J $8b^2$
K $8b^4$

15 $2x^2(3x + 4xy) = (?)$

A $6x^2 + 8x^2y$
B $3x^3 + 4x^3y$
C $6x^3 + 4x^3y$
D $6x^3 + 8x^3y$
E $6x^3 + 6x^3y$

16 Solve $R = \frac{K}{\pi d}$ for d.

F $d = \frac{\pi R}{K}$

G $d = \frac{KR}{\pi}$

H $d = \frac{\pi K}{R}$

J $d = \frac{K}{\pi R}$

K $d = \pi KR$

17 Two of a student's test marks are 68 and 84. A third mark is at least 40. What is his lowest possible average for the three tests?

A 40
B 58
C 62
D 64
E 76

18 When factored, $4a^2 + 12ab^2 = (?)$

F $4a(a + 3b^2)$
G $4a(a + 12b^2)$
H $4ab(a + 3b)$
J $4ab(a + 12b)$
K $4a^2(1 + 3b^2)$

19 A boy who has q quarters and d dimes buys p pencils at 5 cents each. How many cents does he have left?

A $q + d - p$
B $q + d - 5p$
C $25q + 2(d - p)$
D $25q + 10d - p$
E $25q + 10d - 5p$

Go on to the next page.

20 If $3x^2 + bx + 1 = 0$ when $x = 1$, what is b?

F -4
G -1
H 1
J 4
K It cannot be determined from the information given.

21 Which of the following expressions is equal to $(1 + x)(1 + y)$?

A $x + y$
B $1 + x + y$
C $1 + xy$
D $x + y + xy$
E $1 + x + y + xy$

22 $(-2)^3(-3)^2 = (?)$

F -72
G -54
H 36
J 54
K 72

23 If $\frac{x}{3} - 1 = \frac{x}{5} + 2$, then $x = (?)$

A -15
B $-\frac{2}{3}$
C $\frac{3}{2}$
D 15
E $\frac{45}{2}$

24 Factor $3x^2 - 4x - 4$

F $(3x - 2)(x + 2)$
G $(3x + 2)(x - 2)$
H $(3x + 1)(x - 4)$
J $(3x - 4)(x + 1)$
K $(3x - 4)(x - 1)$

25 What is the result when $x^3 - x^2 - 17x + 20$ is divided by $x^2 + 3x - 5$?

A $x - 4$
B $x - 2$
C $x - 1$
D $x + 1$
E $x + 4$

26 If $a = 3$ and $b = 2$, then $\frac{ab^3}{(a - b)^2} = (?)$

F $\frac{18}{5}$
G $\frac{24}{5}$
H 18
J 24
K 216

27 What value of x, when substituted in $\frac{1}{x - 2}$, will make this fraction meaningless?

A -2
B 0
C 2
D Any number between -2 and 0
E Any number between 0 and 2

28 $\frac{5a^2b^2}{4} \div \frac{10b^2}{3a^2} = (?)$

F $\frac{3b^4}{8}$
G $\frac{3a^4}{8}$
H $\frac{25b^4}{6}$
J $\frac{25a^4}{6}$
K $\frac{3}{8a^2b^2}$

Go on to the next page.

29 The statement $x - 6 \leqq 6$ implies that

A $x \geqq -36$
B $x \leqq 0$
C $x \leqq 12$
D $x \leqq 36$
E $x \geqq 12$

30 An automobile is moving at r miles per hour, and an airplane is moving three times as fast. How many hours will the plane require for a 500-mile flight?

F $\frac{1500}{r}$

G $\frac{500}{3r}$

H $\frac{3r}{500}$

J $500 - 3r$

K $1500r$

31 If $4x + 5y = 13$ and $2x + 3y = 7$, then $x =$ (?)

A -2

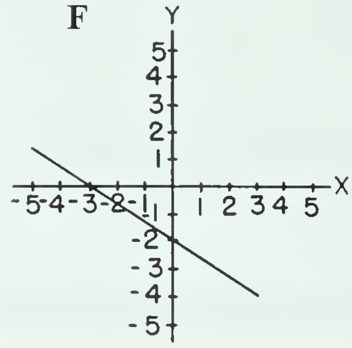
B -1

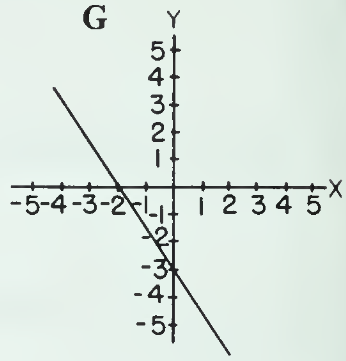
C $\frac{1}{2}$

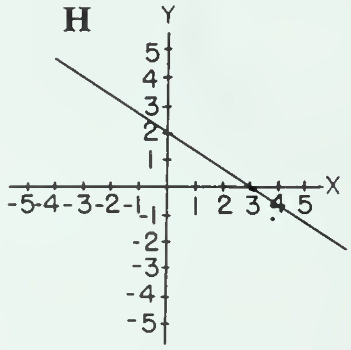
D 2

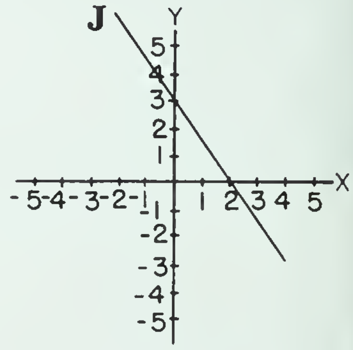
E 4

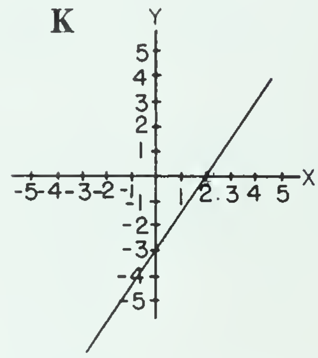
32 Which of the following is the graph of $2x + 3y = 6$?

F


G


H


J


K


33 If x is a real number, what are **all** the values of for which $x^4 + 16$ is a positive number?

A All x greater than -2
B All x greater than zero
C All x greater than 2
D All x between -2 and 2
E All values of x

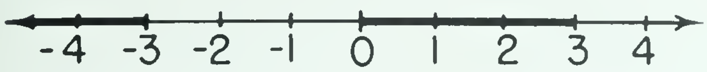
Go on to the next page

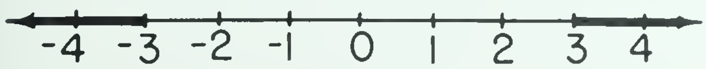
34 Solve the equation $x^2 + 10x - 24 = 0$ for x.

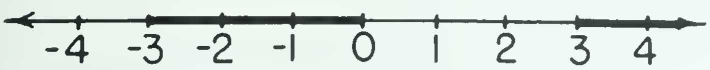
F $x = 12$ and $x = -2$
G $x = 12$ and $x = 2$
H $x = 6$ and $x = -4$
J $x = 6$ and $x = 4$
K $x = -12$ and $x = 2$

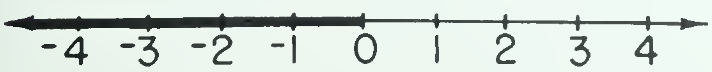
35 On which of the following number lines does the heavy line represent all numbers x such that $-3 \leqq x \leqq 3$?

A 

B 

C 

D 

E 

36 If $y = \frac{1}{x}$ and x is greater than 0, which of the following statements is true?

F As x increases, y increases.
G As x increases, y decreases.
H As x decreases, y decreases.
J When x is greater than 1, y is greater than 1.
K When x is less than 1, y is less than 1.

37 What number must be added to $x^2 - 6x + 4$ in order to make it a perfect square?

A -4
B 0
C 2
D 5
E 32

38 For what values of x is $\frac{x}{6} = \frac{1}{2}(x - 3) - \frac{x}{3}$ a true statement?

F 0 only
G 3 only
H 0 and 3 only
J All values
K No value

39 Solve the formula $E = \frac{ar}{a + r}$ for r.

A $r = \frac{aE}{a - E}$

B $r = \frac{aE}{a + E}$

C $r = aE - a + E$

D $r = aE - a - E$

E $r = a - \frac{E}{a - E}$

40 If x is greater than 3, which of the following is the smallest?

F $\frac{3}{x}$

G $\frac{3}{x + 1}$

H $\frac{3}{x - 1}$

J $\frac{x}{3}$

K $\frac{x + 1}{3}$

Look over your work on this test.